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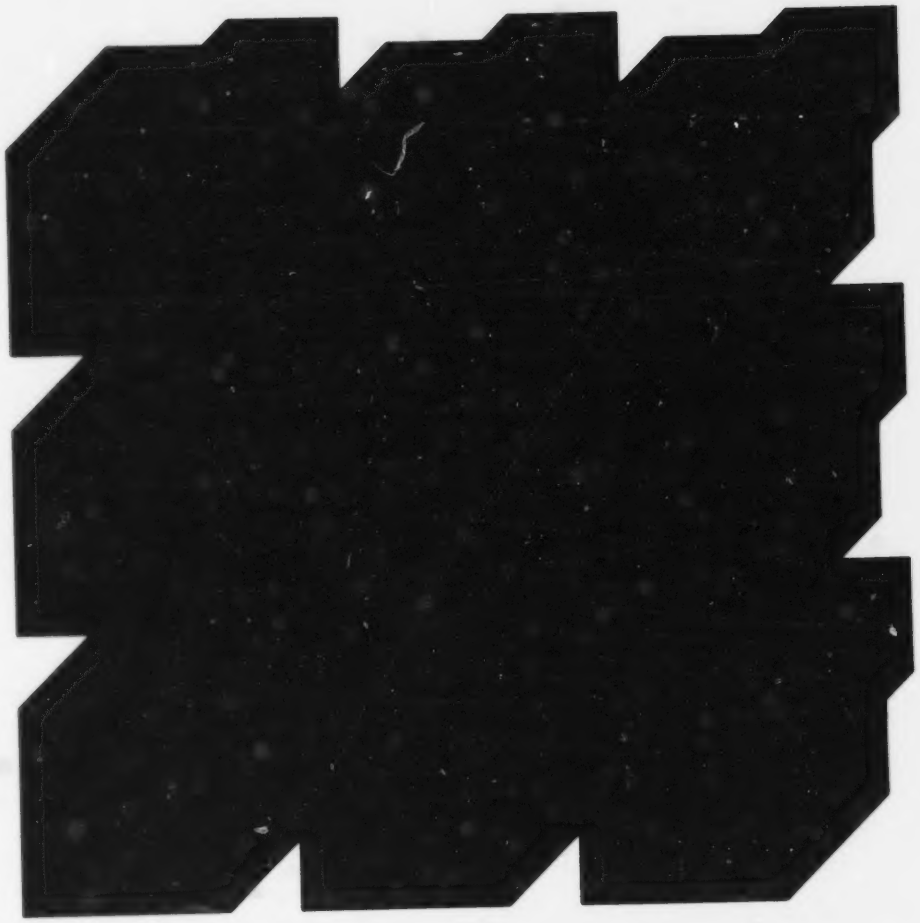
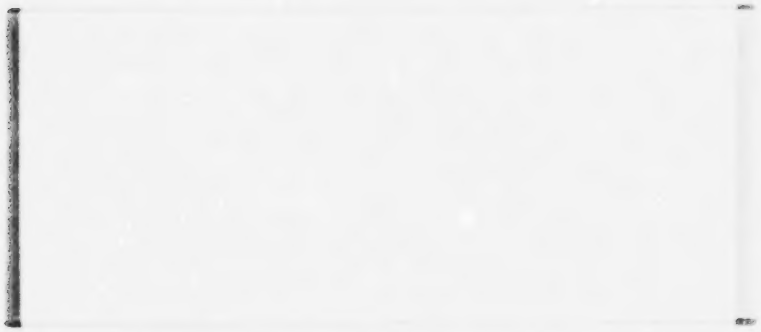
DISCUSSION PAPER NO. 249

The Supply of Oil and Natural Gas Discoveries in Alberta

By: Jacques Jobin





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SOMMAIRE

Cette étude tente de donner une idée de la quantité de pétrole et de gaz naturel à être découverte en Alberta. C'est une version révisée et améliorée du document no. 223 : L'offre de découvertes de pétrole et de gaz naturel en Alberta.

Après avoir fait une revue de la littérature qui utilise des méthodes économétriques pour estimer cette quantité, l'auteur, s'inspirant des études américaines et canadiennes, développe deux variables pour estimer les découvertes : d'une part, une variable économique, le rendement net (net back) et d'autre part, une variable "géologique", les découvertes cumulées. Cette méthode conventionnelle donne des résultats très décevants. L'auteur tente alors d'autres avenues en omettant ou introduisant des variables pertinentes, mais sans succès.

Dans la deuxième partie du travail, qui peut être lue indépendamment de la première, une autre méthode est tentée : elle consiste à estimer les découvertes en estimant et utilisant la courbe de coût marginal de découvertes. Comme on le sait, cette courbe, à long terme, peut être définie comme une courbe d'offre de découvertes. Cette méthode, une fois les données désagrégées, donne des résultats intéressants : brièvement, les résultats du modèle sont qu'il reste du gaz naturel en quantités très importantes alors que pour le pétrole, toujours d'après le modèle, on aurait découvert jusqu'à maintenant les deux tiers des ressources disponibles en Alberta. Tout en considérant que l'ajustement ("fit") de l'équation est un peu faible, si on compare ces résultats avec les prévisions de l'Office National de l'Energie dans son hypothèse optimiste pour le pétrole, celles-ci apparaissent pessimistes et pour le gaz naturel, elles sont presqu'identiques.

ABSTRACT

This study attempts to estimate the quantity of undiscovered oil and natural gas remaining in Alberta. It is a revised and updated version of " 'offre de découvertes de pétrole et de gaz naturel en Alberta" Discussion Paper no. 223.

After reviewing the literature that uses econometric methods to estimate this quantity, the author works from U.S. and Canadian models to develop two variables for estimating the undiscovered reserves: an economic variable, known as net back, and a "geological" variable, the cumulative discoveries to date. This conventional method, however, gives very disappointing results. The author then tries other avenues, by omitting or introducing relevant variables, but again obtains poor results.

The second part of the paper introduces an entirely new procedure and can be read independently of the rest. It estimates the undiscovered oil and gas by estimating and using a marginal cost curve for discoveries. As we all know, this curve can be defined over the long term as a discovery supply curve. When the data are broken down, this method produces interesting results. Briefly, the results of the model are that there remain significant quantities of undiscovered natural gas, but as far as oil is concerned, again according to the model, about two-thirds of the total reserves in Alberta have already been discovered. While the fit of the equation is not ideal, the National Energy Board's oil forecasts seem pessimistic compared to the author's results, but reasonable for gas.

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INTRODUCTION

This work forms part of the background for a study launched in the fall of 1980 by the Economic Council of Canada on the economic implications for western Canada and the rest of the country of the spiralling energy prices that followed the two oil price shocks in 1973 and 1979.

Essentially, we have estimated the supply curve for oil and natural gas discoveries in Alberta, since this curve and the conditions under which it shifts (or rotates) must be known to determine the impact of higher resource prices on the economy. This paper therefore answers a fundamental question: how much oil and natural gas is left in the West and at what price (or cost) is it available? Natural-resource economists are familiar with the need to link a price to the available quantity of a natural resource since, unlike other goods, it is not available in unlimited quantities, even under perfect competition. If the price of this resource is fixed, the quantity available is eventually depleted because the supply curve involves an increasing marginal cost.

In Canada, because imported oil was cheaper, this industry was protected from foreign competition between 1961 and 1973 by the 1961 National Policy, which created a captive market west of the Ottawa River for Canadian oil. Since 1973, many interventions by

the federal and provincial governments have made analysis of this Canadian industry more complex.

The first part of this paper reviews all of the literature that attempts to estimate the supply curve for oil and/or natural gas by econometric or other methods. This first step is quite thorough, to assist us in developing our own approach, based on a profit concept: net back (the data used in this chapter are presented in Appendix 1). We study the results obtained, then introduce another theory, based on estimating the marginal cost curve (the relevant data are presented in Appendix 2). In both cases, results are produced for each resource separately and for both combined. These results are then used to estimate probable discoveries of oil or natural gas in coming years.

Chapter 1: Previous Studies

1.1 Review of the Literature

1.1.1 The U.S. Literature

The first econometric study of the oil and gas industry was conducted in the United States: Fisher (1964) estimated exploration supply and the supply of new oil discoveries for 1946-1954. The author made a distinction between these two supply curves:

> "...because the total of new oil discoveries is the product of three terms: the number of wildcats drilled; the fraction of such wells that are producers; and the average size of field discovered per producing wildcat. The sensitivity of the product depends on the sensitivity of all three factors to economic incentives rather than solely on the sensitivity of the first of them". (p. 7)

Fisher viewed the industry's decision to explore as a process in which the inputs are economic stimuli and natural geological conditions, and the outputs are the number of test wells drilled and the results of this drilling. Faced with a lack of data on the drilling projects considered, Fisher measured the conditions accepted by the industry through three lagged variables: the success rate (the number of productive wells over the number of test wells), the number of barrels of oil from each productive test well and the number of Mcf of natural gas in each productive test well. The lack of data on profits and anticipated costs

limited Fisher to a single economic stimulus, the price of oil. Since the data were broken down by district (five in all), Fisher reorganized them into a cross section regression and a time series.

The author sees an important role for the three lagged variables: they constitute the information available to business on the probable returns and on the characteristics of the regions explored, and by channelling the effect of district differentiation, they reveal the influence of the other variables used on the dependent variables. The results of this model, including the standard errors, are:

$$\log W_{jt} = \underset{(1.54)}{8.29} + \underset{(.000724)}{.00862}\, H_{jt} + \underset{(.525)}{2.85} \log P_{jt} + \underset{(.0985)}{.44} \log S_{jt-1} - \underset{(.304)}{.941} \log F_{jt-1} - \underset{(.119)}{.563} \log N_{jt-1} \qquad R^2 = .84$$

$$\log F_{jt} = \underset{(.651)}{1.99} + \underset{(.116)}{.581} \log F_{jt-1} - \underset{(.0407)}{.15} \log S_{jt-1} + \underset{(.0448)}{.083} \log N_{jt-1} - \underset{(.0000375)}{.000106}\, D_{jt-1} + \underset{(.000283)}{.00059}\, H_{jt} - \underset{(.234)}{.356} \log P_{jt}$$

$$R^2 = .73$$

$$\log S_{jt} = 5.35 + .777 \log S_{jt-1} + .692 \log F_{jt-1}$$
$$(1.73) \quad (.113) \qquad\qquad (.329)$$
$$- .489 \log N_{jt-1} - 2.18 \log P_{jt}$$
$$(.137) \qquad\qquad (.631) \qquad\qquad R^2 = .85$$

where j: an index describing the district

t: an index describing the year

W: the number of new test wells

F: the success rate

S: the average size of oil discoveries by productive test well (in thousands of barrels)

N: the average size of gas dicoveries by productive test well (in millions of cubic feet)

D: the average depth of new wells (in feet)

H: Geophysical teams and preliminary drilling (in team-years)

P: Wellhead price of oil (in constant dollars 1947-1949).

Given the nature of the data and the complexity of the oil industry, the author is careful to stress that the magnitude of the coefficients is more important than the specific values. The price elasticities of the exploration effort, the success rate and the size of the discoveries are 2.85, -.36 (non-significant difference from zero) and -2.2 respectively, for a total price elasticity of .31. An increase of one percentage point in the price of oil produces only a .31 point increase in new discoveries. With these results in hand, the author then rationalizes the sign of the negative coefficients by pointing out that at a

given price, operators retain a list of small, low-risk reservoirs that are unprofitable at current price levels but would become profitable following a price increase. The author concludes:

> It follows that a rise in price induces a decrease in average size associated with an increase in certainty which partially offsets the risk increase which would otherwise occur. The effect is a short-run one and is restricted to price rises. (p. 23)

Fisher was the first to construct an econometric model of the oil industry. The criticisms that follow should therefore be weighed against the significant contribution his pioneering work made to the subject.

Interpreting elasticities is a difficult task. As with any regrouping of cross-section and time-series data, short-term (time-series) and long-term (cross-section) elasticities are lumped together. The explanations of the signs of the coefficents would seem to indicate that these are short-term elasticities, but the model fails to specify this. The lumping of data has another drawback as well, since the author fails to differentiate districts. One district alone accounted for 82.7 per cent of all discoveries in the United States in 1950.

Interpreting the coefficients and their sign after obtaining the results would not be a wise step. The model's development should include the expected sign of the coefficents, which the model then verifies; Fisher rationalizes the sign of the coefficents after

obtaining the results. Similarly, the author does not explain why he divides the new discoveries into three groups but simply states this fact.

The model overlooks three inherent problems in studying the oil industry: resource depletion, growth of marginal exploration costs and the industry's special tax treatment. Natural gas played a negligible role in U.S. energy supply between 1946 and 1954. The author tested the natural gas price coefficient and in no case did it differ significantly from zero. Since that time, however, natural gas has become a major source of energy and any model covering a more recent period should include this variable.

Fisher concludes:

> ...our findings do cast considerable doubt on policy arguments resting on the assertion that oil discoveries are highly sensitive to economic factors. To the extent that this argument depends on evidence as to high sensitivity of wildcat drilling -- or other exploratory and development drilling -- it substantially overstates the case. (p. 39)

The price elasticity of .31 is actually small. Other authors who have used this same model for a longer period have obtained a higher price elasticity.

Erickson and Spann (1971) used Fisher's model for the 1946-1958 period. Their most important change was to add natural gas variables to Fisher's oil variables. The model produced the

following results: the price elasticity for the supply of oil discoveries is the sum of three elasticities (1.48 for the exploration effort, -.23 for the success rate and -.42 for the average size of oil discoveries) a total of .83, more than double Fisher's results. The price elasticity for the supply of natural gas discoveries was computed in the same manner, giving .69 (.35, .01, .33).

Eyssel (1978) updated Fisher's model for the period 1946-1970. He concluded:

> "It is difficult to generalize about the extent to which crude and natural gas are joint products. (...) However, in some provinces distinct gas and oil reservoirs are found with no particular pattern within the area. The probability of finding gas rather than oil increases with depth, and below about 15,000 feet, gas is a virtual certainty. The probability of finding gas also increases as the degree of low grade metamorphism of reservoir rocks increases." (p. 21)

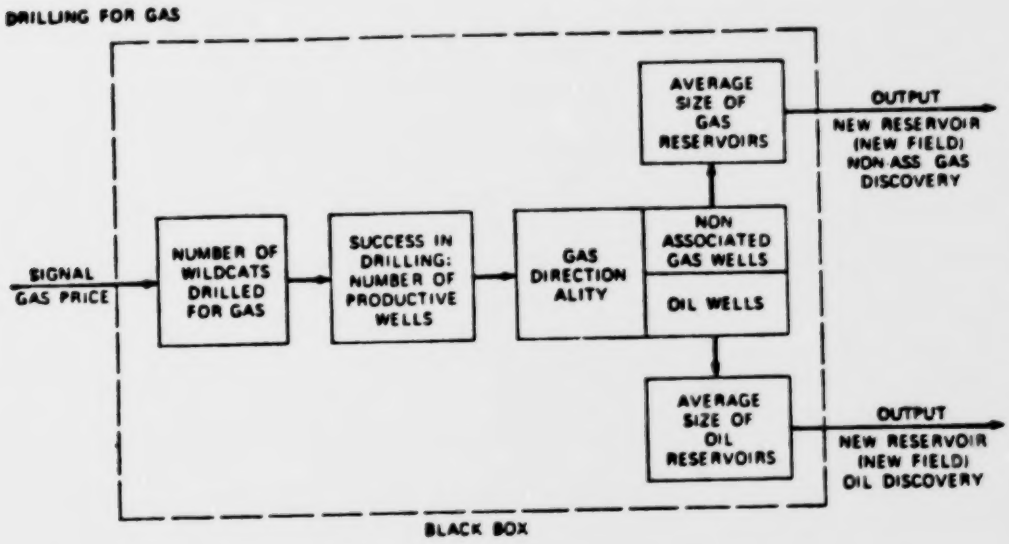
The author continues, stressing "...every successful new field wildcat reported by the AAPG through 1970 for North Dakota has been a crude producer: thus oil can be a separate product." (p. 21)

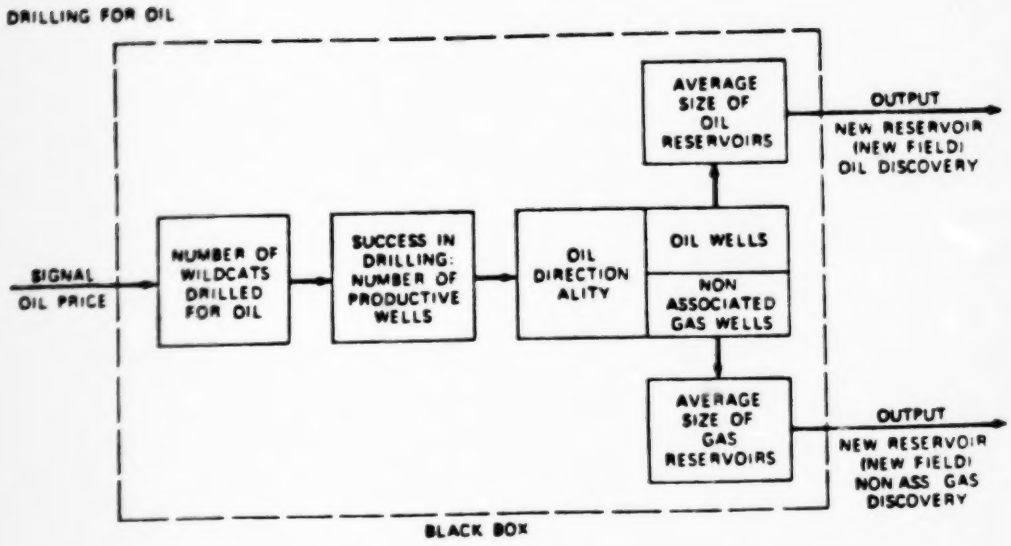
The author thus decides to eliminate "any consideration of gas and its price from his analysis." The price elasticity of the exploration effort is 1.84. Those for the success rate and size of discoveries have a positive and non-significant coefficient of

.22 and .37 respectively. The total price elasticity is therefore 2.43.

The poor quality of Eyssel's work severely limits its usefulness. The author confuses the mere presence of natural gas with actual production. Although some states may produce only oil, this is no proof the reservoirs contain only oil. The natural gas may be burned off if there are no pipelines and/or customers. This is the most serious fault in his work, particularly since the study is recent (1978) and since natural gas greatly expanded its share of the energy market during the period analysed (1946-1970).

Khazoom (1971) takes an entirely new approach. He bases his model on the fact that some discoveries of gas and/or oil can be traced to an increase in the price of gas and/or oil. The approach is illustrated in the two diagrams below:





The three variables used by Fisher are evident in the black box: the number of wildcats, the success rate and the average size of discoveries. The author explains his model in this manner:

> We seek to discover the relationship between the decision triggered by the signal at one end of the black box and the final product at the other end of the box. (p. 56)

The basic assumption of this approach is that the producer seeks the largest production volume possible for each additional dollar spent on exploration. The probable sites of discoveries are known, so those promising the greatest return are drilled first, and so forth. If the producer has a thorough knowledge of the various sites, supply can only be increased through an increase in price. Khazoom established his regions on the basis of geological homegeneity, rather than the geographic homogeneity used by Fisher. However, he does use the same method of lumping data into cross sections and time series. Each dependent variable is

estimated by a linear equation and a non-linear (quadratic) equation. The model is written:

$$ND_t = -\mu_0 + \mu_1 \sum_{i=1}^{2} C_{t-i} \pm \mu_2 \sum_{i=1}^{2} P_{O_{t-i}} + \mu_3 \sum_{i=1}^{2} P_{L_{t-i}} + \mu_4 \sum_{i=1}^{2} ND_{t-i} + \omega_t$$

$$ND_t = -\mu_0' + \mu_1' \sum_{i=1}^{2} C_{t-i} \pm \mu_2' (\sum_{i=1}^{2} C_{t-i})^2 \pm \mu_3' \sum_{i=1}^{2} P_{O_{t-i}} \pm \mu_4' (\sum_{i=1}^{2} P_{O_{t-i}})^2 + \mu_5' \sum_{i=1}^{2} P_{L_{t-i}} \pm \mu_6' (\sum_{i=1}^{2} P_{L_{t-i}})^2 + \mu_7' \sum_{i=1}^{2} ND_{t-i} + \omega_t'$$

$$XR_t = -\pi_0 + \pi_1 C_t - \pi_2 P_{O_t} + \pi_3 P_{L_t} + \pi_4 ND_{t-1} + \pi_5 XR_{t-1} + \xi_t$$

$$XR_t = -\pi_0' + \pi_1' C_t - \pi_2' C_t^2 - \pi_3' P_{O_t} + \pi_4' P_{O_t}^2 + \pi_5' P_{L_t} - \pi_6' P_{L_t}^2 + \pi_7' ND_{t-1} + \pi_8' XR_{t-1} + \xi_t'$$

where ND: New discoveries of natural gas,

XR: Extensions and revisions of natural gas,

C: Ceiling price of natural gas,

Po: Price of oil,

Pl: Price of liquified gas (Propane, Butane),

ξ, ω: Residuals

μ, π: Coefficients

As we can see, the lagged variables are computed over the average of the two preceding years in the new discoveries equations. The author estimates operator's reaction time to a change in economic conditions at one year. Extensions or revisions are measured immediately after a change in the price of one of the three components. This is a new, more realistic approach: operators' reaction cannot be immediate. Similarly,

quadratic equations are estimated for the first time, but unfortunately, none of the estimated price coefficients are significant, so their results will not be given here. The results for the linear equations for the 1961-1969 period (with Student values) are:

$$ND_t = -226.07 + \underset{(2.18)}{15.29} \sum_{i=1}^{2} \underset{(2.20)}{C_{t-i}} - \underset{(.26)}{2.69} \sum_{i=1}^{2} Po_{t-i}$$

$$+ \underset{(.56)}{3.98} \sum_{i=1}^{2} Pl_{t-i} + \underset{(19.27)}{.84} \sum_{i=1}^{2} ND_{t-i}$$

$$R^2 = .61$$

$$XR_t = -756.38 + 49.36\ C_t - 30.85\ Po_t + 47.38\ Pl_t$$
$$\qquad\quad (1.93)\quad (1.87)\qquad (.80)\qquad\quad (1.70)$$

$$+ .72\ ND_{t-1} + .49\ XR_{t-1}$$
$$(4.52)\qquad (8.46)\qquad\qquad R^2 = .61$$

Since none of the price coefficients for XR_t are significant, the author abandons this equation and bases the rest of his study on the results obtained for ND_t. Since Khazoom has not transformed these data into logarithms, the elasticities are in absolute values: if the ceiling price for natural gas increases by one cent for just one year, new discoveres of natural gas will increase by 7.645 million Mcf. The author then abandons the Po and Pl variables, and turns to the "time path of discoveries response to gas price." The new discoveries equation then becomes:

$$ND_t = \beta(C_{t-1} + C_{t-2}) + \delta(ND_{t-1} + ND_{t-2})$$

He seeks the total effect: "The total direct (by C_{t-1}) and indirect (by ND_{t-1}) effect of C_{t-2} can be derived by eliminating ND_{t-1} from the equation by means of the same equation lagged one year." (p. 68)

$$ND_t = \beta C_{t-1} + \beta(1 + \delta)\ C_{t-2} + \beta\delta C_{t-3} + \delta(1 + \delta)ND_{t-2} + \delta^2 ND_{t-3}$$

If the ceiling price increases by one cent, the impact will be β million Mcf the following year and $\beta (1 + \delta)$ two years later. The impact after eleven years is shown below, with each line corresponding to a year:

GENERAL EXPRESSION	1	2	3
β	7.645	7.645	8.1%
$\beta (1 + \delta)$	10.846	18.491	19.7%
$\beta (2\delta + \delta^2)$	7.743	26.234	27.9%
$\beta (\delta + 3\delta^2 + \delta^3)$	7.784	34.017	36.2%
$\beta (3\delta^2 + 4\delta^3 + \delta^4)$	6.502	40.519	43.1%
$\beta (\delta^2 + 6\delta^3 + 5\delta^4 + \delta^5)$	5.982	46.501	49.4%
$\beta (4\delta^3 + 10\delta^4 + 6\delta^5 + \delta^6)$	5.227	51.728	55.0%
$\beta (\delta^3 + 10\delta^4 + 15\delta^5 + 7\delta^6 + \delta^7)$	4.694	56.422	60.0%
$\beta (5\delta^4 + 20\delta^5 + 21\delta^6 + 8\delta^7 + \delta^8)$	4.154	60.576	64.4%
$\beta (\delta^4 + 15\delta^5 + 35\delta^6 + 28\delta^7 + 9\delta^8 + \delta^9)$	3.705	64.281	68.3%
$\beta (6\delta^5 + 35\delta^6 + 56\delta^7 + 36\delta^8 + 10\delta^9 + \delta^{10})$	3.29	67.572	71.8%
		94.074	

The impact of a one-cent increase for one year is illustrated in column 1. If the increase remains in force permanently, the impact is that illustrated in column 2. Column 3 gives the impact of a one-cent increase in cumulative percentage, i.e., column 2 over the cumulative total (94.074). Khazoom then makes the following assumption: since the price of natural gas is the equilibrium price, he simulates a 1969 price between 0 and 15 cents greater than the true price for that year, and for each additional cent, he computes the corresponding annual price elasticities for the next eleven years. He discovers that the higher the equilibrium price of gas in 1969, the more the annual price elasticities approach unity.

It is strange that an increase in C_t for a single year would induce an increase of 7.6 million Mcf the following year and 10.8 million Mcf two years later, after the ceiling price has returned to its earlier level. It also seems strange that the impact of this increase would extend over such a long period (15 years). But both facts are the result of incorporating the impact into ND_{t-1}.

This study is ten years old and although the model's forecasting section is no longer valid, Khazoom did introduce a new approach to discoveries supply that differed from Fisher's.

Erickson, Millsaps and Spann (1974) estimated the supply of oil between 1950 and 1968 by estimating the supply of oil reserves. They argued that "the principal economic determinants of desired reserves are the expected price of oil and the user cost of oil reserves. User cost is a measure of the implicit price to the firm of capital embodied in oil reserves..." Lacking a measure of the anticipated price, the authors assumed that it is determined by the current price and the rationing of production. Integrating the industry's special tax treatment, the authors define the concept of user cost:

$$C_t = q_t \left[\frac{r\ (1 - T\gamma) + \delta(1 - T\gamma - Tv)}{1 - T + Td} \right]$$

where CT: User cost,

qt: Exploration costs of supplementary reserves, per barrel,

r: Capital cost,

T: The sector's tax rate,

γ: That part of capital considered current expenditure and, therefore available for write-off,

δ: The depreciation rate of capital stock or reserves,

v: That part of capital which can be written off for tax purposes,

d: The rate of the deduction for depletion.

The authors point out that a full demonstration of this equation is available upon request. Since they have no data for qt, they drop this variable and assume it is constant. The results, with the standard errors, are:

$$\ln R_{t,j} = \underset{(.71)}{1.41} + \underset{(.09)}{.102} \ln P_{t,j} - \underset{(.028)}{.069} \ln C_t - \underset{(.083)}{.067} \ln K_{t,j}$$

$$+ \underset{(.07663)}{.076} \ln K_{t-1,j} + \underset{(.04)}{.9} \ln R_{t-1,j} - \underset{(.269)}{.487} D_1$$

$$- \underset{(.204)}{.125} D_2 - \underset{(.21)}{.152} D_4 - \underset{(.201)}{.095} D_5$$

$$R^2 = .99$$

where t: Year indicator,

j: District indicator,

R: Proven oil reserves (i.e., cumulative discoveries less production) in thousands of barrels,

P: Constant average wellhead price for a barrel of oil (i.e., cumulative discoveries less production),

C: User cost,

K: Texas Shutdown Days,

D: Dummy variables to differentiate districts.

The authors then construct the model on the assumption that C_t and P_t "enter the determination of reserves symmetrically" (p. 470), i.e., that a 10 per cent increase in the price of oil is equivalent to a 10 per cent drop in user cost, and vice versa. They therefore assume that the coefficient affecting C_t and P_t is the same, which gives:

$$\ln R_{t,j} = \underset{(.695)}{1.37} + \underset{(.0262)}{.073} (\ln P_{t,j} - \ln C_t) - \underset{(.079)}{.059} \ln K_{t,j}$$
$$+ \underset{(.0747)}{0.070} \ln K_{t-1,j} + \underset{(.039)}{.9} \ln R_{t-1,j} - \underset{(.25)}{.45} D_1$$
$$- \underset{(.20)}{.11} D_2 - \underset{(.204)}{.14} D_4 - \underset{(.20)}{.08} D_5$$

$$R^2 = .99$$

In the first case, the long-term price elasticity is close to one, while in the second equation it is .76. The authors conclude their study with forecasts to 1985 based on three prices for a barrel of oil ($8, $10, $12) and the elimination of resource depletion.

In this article, the supply of oil is considered equivalent to the supply of oil reserves. There is an ambiguity in estimating the supply of reserves, because as production proceeds, proven reserves are depleted; new discoveries (or adequate technological advance) are necessary to maintain constant reserve levels. In other words, growth of reserves represents new discoveries less output. The authors therefore measure the elasticity of marginal oil discoveries to keep proven reserves at a constant level.

The user cost procedure has two weaknesses: first, it is incorrect to assume exploration costs remain constant throughout the period (1950-1968) and second, this procedure should also take into account the costs of developing reserves, since development is just as vital a step as discovery in the process of increasing oil supply.

Finally, the authors commit a serious error by failing to include natural gas and gas prices into their analysis. As we have already pointed out, natural gas is a frequent by-product of oil discoveries. The quality of this work is therefore seriously compromised.

Nonetheless, the authors have been innovative in defining user cost with the industry's special tax treatment, the first time this appears in the literature.

In his doctoral thesis, Epple (1975) uses a new procedure to estimate the supply of oil discoveries for the 1947-1968 period. His major assumption is:

> "The analysis of the decisions of the firm exploring for oil and natural gas will be facilitated by assuming that there is a well functioning market for oil and natural gas discoveries. When a discovery is made, the firm may exploit the deposit itself, or it may sell the rights to the discovery to another firm. Let P_o and P_g be the after tax price per unit of oil and gas in the ground. These are not wellhead prices but rather the unit values of oil and natural gas in the reservoir prior to the drilling of producing wells. The major inputs in the exploration process are exploratory wells and oil-bearing land." (p. 67)

This approach enables the author to analyse the oil and gas industry at the exploration level, as a problem of a firm maximizing a profit function subject to the constraint of a production function in a competitive market. Epple estimates the production function as follows:

$$(1) \quad \pi = P_o Y + P_g G - CF - RL$$

where P_o: Price of oil in the ground

Y: Quantities of oil discovered

P_g: Price of natural gas in the ground

G: Quantities of natural gas discovered

C: After-tax per-unit (per foot) cost of drilling

R: After-tax cost of renting oil-bearing land

F: Actual drilling

L: Quantity of oil-bearing land used

The production function is:

$$(2) \quad [a\ Y^d + (1-a)\ G^d]^{1/d} = A_1\ F^m\ L^n\ e^{\gamma_1 W}$$

The left member of (2) appears to be in CES form. Yet Epple chooses CET form (Constant Elasticity of Tranformation) to eliminate the constraint in the CES that "d" must be less than one. CET constrains "d" to be greater than one.

The variable "w" measures technical advance. The author has chosen to measure this variable by the cumulative drilling of exploratory wells.

The price of a discovery is definitely the most difficult factor to determine with this approach. Epple proceeds as follows:

$$(3) \quad VPN = b_1\ e^{ht}\ (1 - T + Td)\ (PDxkQ)/(Dx + r)$$

where VPN: Present net value

b_1: The proportion of earnings attributable to exploration (measured by the proportion of costs attributable to exploration)

h: Trend of b_1 over time

T: Corporation income tax rate

d: Rate of deduction for depletion

P: Wellhead price

D: Desired decline rate, the fraction by which production drops from one period to the next

K: Proportion of oil (30 per cent) or natural gas (100 per cent) recoverable from the reservoir

Q: Total quantity in the reservoir

r: Interest rate

x: Production quota

If there is no production quota, x = 1. Remember that this method gives the market price of a discovery.

We should again emphasize the originality of this approach. Earlier works estimated oil and gas supply by econometrically estimating the coefficients of independent variables considered to be determinants in the existence and development of a dependent variable selected to explain the supply of oil and natural gas. But Epple takes a different approach: he estimates the parameters of a production function and the first order conditions to obtain a maximum in the context of a firm maximizing a profit function, and selling its product (new discoveries) in a competitive market, with given factor prices and costs. We have already presented the profit and production functions. After determining the first order conditions and completing a few operations, Epple arrives at:

$$(a) \quad [(a)Y^{d} + (1-a)G^{d}]^{\frac{1}{d}} = AF^{m+\beta}\left(\frac{C}{m}\right)^{\beta} e^{gW}$$

$$(b) \quad \frac{Y}{G} = \left(\frac{1-a}{a}\right)^{\frac{1}{d-1}} = \left(\frac{P_o}{P_g}\right)^{\frac{1}{d-1}}$$

$$(c) \quad \frac{CF}{P_o Y} = m + m\left(\frac{1-a}{a}\right)\left(\frac{G}{Y}\right)^{d}$$

where

$$g = \gamma_1 + \frac{\gamma_2 n}{(\beta_1+1)}, \quad \beta = \frac{n\beta_1}{(\beta_1+1)}, \text{ and } A = A_1 n^{\beta}$$

The author transforms these three equations into logarithms, finds the explicit functions for Y, G and F and estimates the parameters of the equations obtained through the maximum likelihood with the quadratic hill-climbing method.

He obtains the following results:

$$(a) \quad \text{Log } Y = \underset{(1.03)}{2.768} + \underset{(2.74)}{2.869} \text{ Log } P_o - \underset{(2.90)}{1.678} \text{ Log } C_I$$

$$-.0685 \text{ Log } [1. + 8.94 \left(\frac{P_g}{P_o}\right)^{4.15}_{(4.15)}] - \underset{(4.48)}{.672} W$$

$$(.53) \qquad (1.79)$$

$$\text{(b)}\quad \underset{(.70)}{\text{Log } G = -.728} + \underset{(2.74)}{2.869 \text{ Log } P_g} - \underset{(2.90)}{1.678 \text{ Log } C_I}$$

$$-.0685 \underset{(.53)}{\text{Log}} \underset{(1.28)}{\left[.112 \left(\frac{P_o}{P_g}\right)^{4.15}(4.15) + 1\right]} - \underset{(4.48)}{.672 W}$$

$$\text{(c)}\quad \text{Log } F = -3.62 - \underset{(4.53)}{2.678 \text{ Log } C_I} + \underset{(5.08)}{.931} \text{ Log } \underset{(1.33)}{\left[.031 P_o^{4.15}(4.15)\right.} + \underset{(4.4)}{.279 P_g^{4.15}(4.15)\left.\right]} + \underset{(.45)}{.082 W}$$

Oil and/or natural gas discoveries will increase by 2.87 percentage points for each one percentage point increase in price (of discoveries) and will drop by 1.678 points for each one point increase in costs. Since the annual average change in W is .1, the coefficient of W indicates that the discovery supply curve is shifting to the left at a rate of 6.7 per cent a year.

Epple's study is thorough. He takes into account the cost of inputs, the industry's tax treatment, the depletion of discoveries and the presence of joint production in the industry. His argument is difficult to assail.

1.1.2 The Canadian Literature

Eglington (1975) produced a doctoral thesis on "The Economics of Industry Petroleum Exploration". He argues that recoverable reserves have a market price. Unlike Epple, who estimates the parameters of a profit function under the constraint of a production function, Eglington chooses variables relevant to the derivation of a supply function; the use of each of these variables is thoroughly justified and this justification forms the very core of his thesis. The same applies for the use of the market-price formula for reserves. There is no need to review these justifications here, but we will retain the reserve price formula and the oil and gas supply function from Eglington's work. The author divides the joint product by the exploration intent, with the double intent lumped in with the oil exploration intent.

It should be remembered that Epple estimates the price of discoveries and Eglington the price of recoverable reserves, with measurement simplified by assuming that the ratio of production to recoverable reserves is constant. Eglington must include the production costs in his reserve price formula. This price is written:

$$P_{R,o} = \frac{1}{i.T_o}.(1-e^{-iT_o}).\ (1-\tau).(P_o.(1-C_5)-C_1)-(1-D_1.\tau).C_4$$

$$- \frac{(1-\tau).C_2 + F_1.C_3}{T_o.Q_o} \ . \frac{e^{-i\Delta_o}}{1-D_2.\tau}$$

where:

$Pr_{,o}$: Demand price of a reservoir anticipated by the explorer

C_1 : Operating cost in dollars per barrel

C_2 : Development cost

C_3 : Surface equipment cost

C_4 : Leasing cost

C_5 : Royalty rate

Δ : Lag between discovery and production

i : Interest rate

T_o : Production period

P_o : Wellhead price

D_1, D_2: Dummy variables to adjust for changes in tax rates

Q_o : Annual productivity

F_1 : Capital deductions for equipment

τ : Actual rate of taxation

The term Δ applies only to natural gas, since for oil, $\Delta = 0$ for the entire period. In addition to the ratio of production to constant reserves, two other assumptions are necessary for this formula to be valid: "variable costs are constant over the production life of the pool, and [...] variable inputs are not a decision instrument." (p. 64)

The equations defining the supply of oil discoveries by the oil discovery intent (or double intent) and of natural gas discoveries by the gas discovery intent are:

$$\ln R_{o,o,T} = 26.2 - 4.1\ln P_{X,T} + 2.3\ln P_{R,o,T-1} + 0.01\ln P_{R,G,T-1} + 0.44\ln X_{o,o,T-1} - 0.15\ln X_{G,G,T-1} + 0.87\ln S_{o,o,T} - 0.15\ln S_{G,G,T}$$

$$\ln R_{G,G,T} = 14.9 - 1.6\ln P_{X,T} + 0.4\ln P_{R,o,T-1} + 0.3\ln P_{R,G,T-1} + 0.7\ln X_{o,o,T-1} + 1.0\ln X_{G,G,T-1} - 0.09\ln S_{o,o,T} + 1.1\ln S_{G,G,T}$$

where:

$R_{o,o}$: oil discoveries at t (thousands of barrels)

P_X : cost of a well, "any intent"

$P_{R,O}$: price of recoverable oil reserves in the ground

$P_{R,G}$: price of recoverable gas reserves in the ground

$X_{o,o}$: success rate for the oil exploration intent (G: natural gas)

X_o : drilling rate for oil intent (G: natural gas)

$S_{o,o}$: anticipated size of oil reservoir (G: natural gas) for the oil intent

Since complete data were not available before 1952, Eglington studies the 1952-1970 period. Unlike Fisher, he considers the success rate an independent variable that influences discoveries.

Problems with the exogeneity of this variable will be discussed later. The elasticities obtained are short term in both cases. He also estimates a non-logarithmic equation which gives price elasticity for oil discoveries of 1 compared with 2.3 for the logarithmic equation. The author therefore argues that the transformation into logarithmic form is a poor approximation of the supply curve.

Uhler (1976) did not specifically estimate the supply of oil but rather the supply of oil and gas reservoirs through a behavioural production function. This author views the process of discovering a reservoir in a limited area as a Poisson distribution. As expected, the author finds that the rate of discoveries, under constant prices, increases initially, peaks and then declines. Uhler broadens his analysis, with the same procedures, to cover oil reserves in a given region. He finds that as the region is developed, exploration costs rise. The author separates oil discoveries from gas discoveries by the exploration intent, stressing that although a natural gas exploration intent may result in discovery of oil, the process remains a Poisson distribution.

The results are obtained through the maximum likelihood. The elaticities in relation to exploration effort are 1.05, 1.18 and 1.1 for discoveries of oil, natural gas and the joint product respectively. This study is of little use to those examining aggregate oil supply. Since the author did not take into account

the price of oil (or natural gas), the exploration effort is analysed in the context of discovery depletion in a given region. A producer will increase his exploration effort when he expects prices for his product to rise, not when discoveries are likely to be available. Second, it is clear that under conditions of equal return, a producer will consider all potential regions. If a distant region promises a better return than a neighbouring region, the explorer will turn his attention in that direction. It is therefore unfortunate the study did not cover all of Canada.

In 1979, the same author produced another study, "Oil and Gas and Finding Costs", under the auspices of the Canadian Energy Research Institute. In this study, Uhler estimates discoveries of oil and gas separately by dividing inputs by the exploration intent (with the double intent included under oil intent). He believes that discoveries are a function of three inputs (drilling, land leasing and geophysical teams) and of cumulative discovered reserves since the start of the period. Stressing that all oil discoveries in Alberta since 1947 have been in five large fields, Uhler believes the discoveries follow the path of "high initial discovery rates followed eventually by a sharp decline leading to quite low discovery rates." The inputs are determined by a translogarithmic funtion (with three inputs and two outputs); the cumulative discovered reserves are estimated by an exponential function. The total form of the discoveries equation becomes:

$$\ln Y_o = \ln h(X_o) + \ln (F_i (R_{oi}))$$

where:

Y : rate of oil or natural gas discoveries

i : reservoir indicator

X : inputs vector

R : cumulative reserves

o,g : oil or gas indicator

Through $\ln h(X_o)$, the author expresses a translogarithmic function that estimates the share of each input in the function. Function Fi is defined as the exponentiai form. Using the translog function, the author examines the substitution elasticities, obtains an elasticity of -.69 for drilling, -1.01 for land leasing and -1.25 for geophysical teams. Including the wellhead price of oil and natural gas, the authors find the following results:

$$\ln Y_o = \ln h(X_o) \underset{(.546)}{-1.689} \underset{(.0561)}{-.367 R_o} \underset{(.505)}{-.552 \ln p_o}$$

$$\ln Y_g = h(X_g) + \underset{(2.34)}{1.327} + \underset{(.0245)}{.0748 R_g} - \underset{(.0054)}{.0016 R_g} \underset{(.492)}{-.536 \ln p_g}$$

Both price coefficients differ non-significantly from zero, thus the wellhead price (Po or Pg) of either product has no influence

on the quantity of new discoveries. A negative relationship between wellhead prices and the discovery rate would have been surprising.

We will now analyse the non-econometric literature, including two papers from the National Energy Board (NEB): "Canadian Oil: Supply and Requirements, September 1978", and "Canadian Natural Gas, Supply and Requirements, February 1979". Both studies are based on testimony, submissions and statements from all interested parties, and are not econometric. The results are based on four assumptions:

- economic growth of 4.2 per cent a year;
- the real international price of a barrel of oil remaining constant at the 1977 level, i.e., $14.90 in 1977 Canadian dollars;
- Canadian oil price parity with the world price in 1982 (which assumes an increase of two dollars a year);
- the price of natural gas remaining at 85 per cent of the price for oil equivalent.

Time has shown these assumptions to be mistaken, particularly the assumption that the real international price of oil would remain stable. The Department of Energy, Mines and Resources (EMR) revised its forecasts in the paper "Canadian Oil and Gas Supply - Demand Overview" in November 1979. The new assumptions are:

- the ratio of the international landed price for oil to the U.S. wholesale price would increase by 2 per cent a year;

- the Canadian price would reach parity with the world price in 1986 ($4 yearly increases between 1979 and 1982, and $5 between 1982 and 1986);
- the price of natural gas would remain pegged at 85 per cent of the price for oil equivalent.

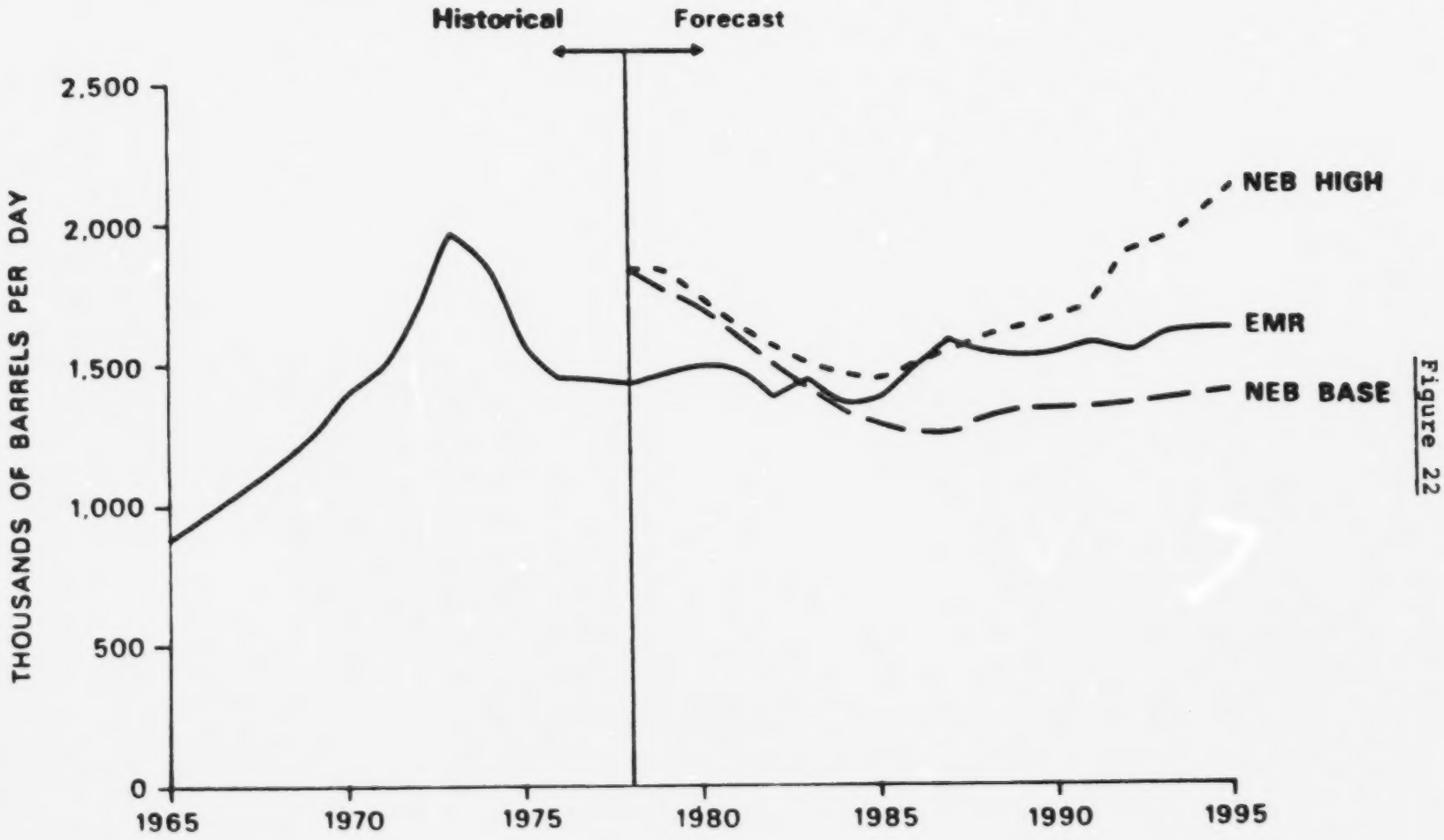
This paper served as the basis for the National Energy Program (NEP). The paper focuses primarily on the total energy demand aspect in Canada for 1980-2000, and therefore includes coal, electricity and nuclear power. Oil and gas supply in fact occupies very few of the study's 100 pages. EMR constructed several price scenarios and measured the oil and gas supply for only one: the Canadian price reaches world levels at $45.25 in 1986. The results are:

Revised EMR Balance, Parity in 1986

	Price			Domestic Output (EMR Price) (T BTU)	
	NEP	EMR		Oil	Gas
1978	12.10	12.10	1978	3028	2481
1979	13.25	13.25	1985	2871	2779
1980	15.75	17.25	1990	3214	2255
1981	18.25	21.25	1995	3381	2087
1982	20.25	25.25			
1983	22.25	30.25			
1984	26.12	35.25			
1985	30.60	40.25			
1986	37.00	45.25			

The table is illustrated in Figure 22.

COMPARISON OF OIL SUPPLY PROJECTIONS



Source: NEB, Oil Report, September 1978
EMR, '1986 Parity' Case

Figure 22

Assuming that growth is sustained, these figures indicate a price elasticity very close to zero. The authors themselves can best explain this pessimistic view:

> Because there is so little reliable evidence about the magnitude of supply price responses, however, this report takes a cautious approach and includes only probable modifications of known projects together with a modest expansion of heavy oil production and upgrading. This is clearly an area where a range of outcomes is possible depending, in part, on non-economic policy initiatives. (p. 47)

The authors stress that the main contributions to the increase in the supply of oil will come from non-conventional oil projects, while conventional oil will play a shrinking role.

Slagorsky (1980) reviews these conclusions and estimates the supply of five types of oil for 1980-2000: conventional oil, frontier oil, oil sands, heavy crude and advanced recovery. In each case, he constructs a supply price for 1980 "that will cover all the expenditures incurred by the producer (i.e., exploration, capital and operating outlays) and also includes a return on investment." (p. 4) This supply price does not include taxes, royalties and transportation costs. He establishes a spread in these forecasts (of prices and production) and by averaging two scenarios, determines the base forecast. The price spreads are very large: the smallest is $6.10 and the largest (for advanced recovery) $12.00 a barrel. He obtains very large spreads for 1985-2000 as well. Between 1980-1985, production fluctuates

Figure 4: Canadian Oil Supply and Demand Projections
Slagorsky's Projections

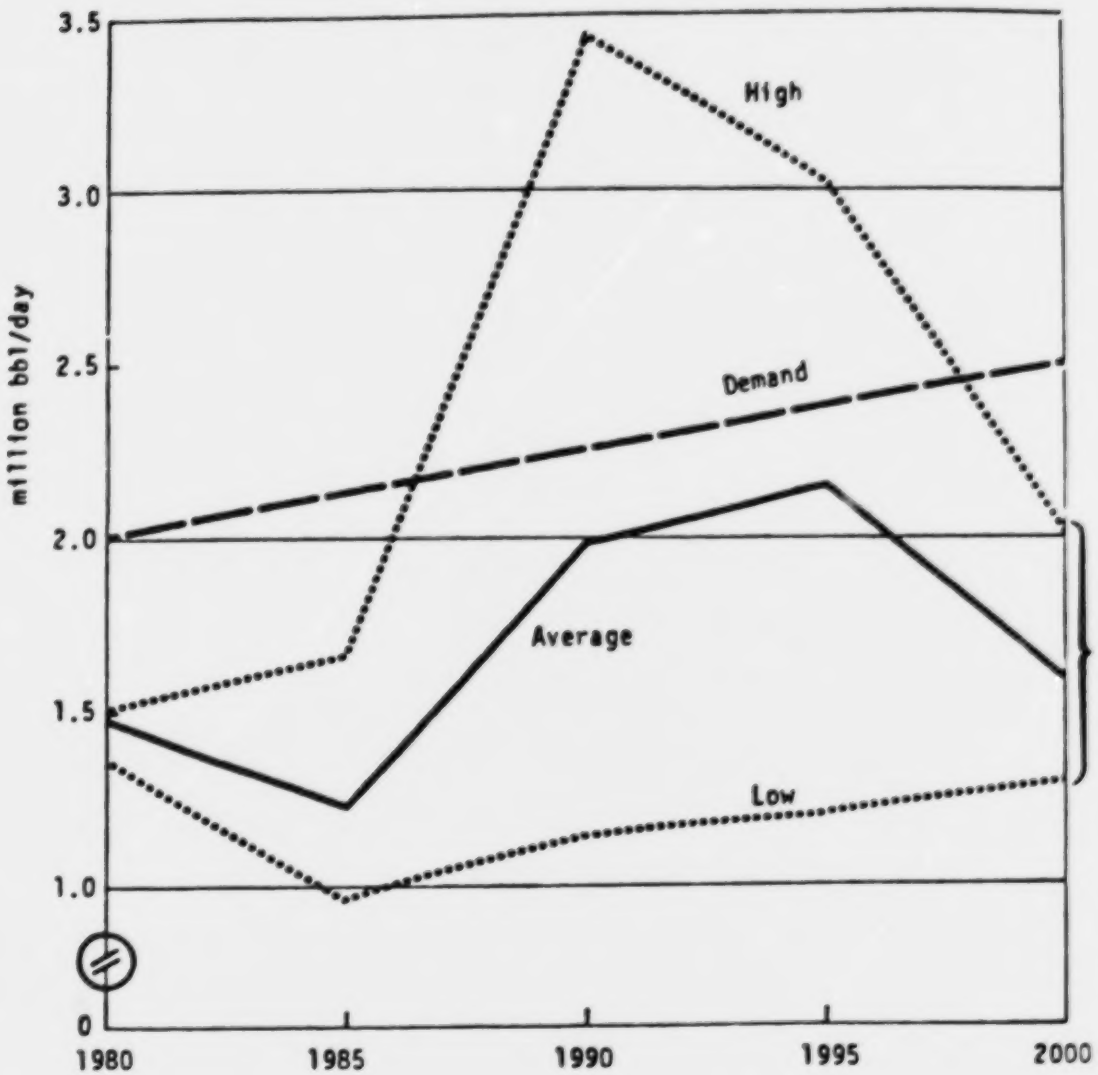


FIGURE 4: Canadian Oil Supply and Demand Projections.

SOURCE: Table 1. Demand projection is adopted from NEB, Gas Export Report, 1979 (the medium demand case, growing at the rate of 1 percent per year).

between 1.4 and 1.5 million bbl/day for 1980 and .9 and 1.7 million bbl/day for 1985. The largest spread occurs in 1990, 1.2 to 3.4 million bbl/day, as shown in Figure 4.

These results are of limited value for two reasons. First, as the spread between high and low assumptions in this type of study grows, the results become increasingly reliable but their usefulness declines. Second, the price assumption is somewhat facile: the author again constructs a price spread for each type of oil, which he then uses to measure probable supply, specifying that "taxes, royalties and transportation charges would have to be added to the supply prices in order to compare the cost of oil from various sources at the main consuming areas of Montreal and Toronto." (p. 4) This approach is based on scenarios that estimate the behaviour of the major variables for predicting oil supply, but the author disregards the presence of natural gas in Canada. Nonetheless, dividing oil into five types is definitely a step in the right direction.

The final paper analysed in this review of the literature is the National Energy Program (NEP). The section on the program's impact estimates future oil and gas supply, including the effects of the program itself. The results for oil are:

	1979	1985	1990
	(thousands of bbl/day)		
Conventional	1388	914	713
Non-conventional	102	326	733

The Canadian government expects the additional reserves of conventional oil from provincial lands to be too small to offset the depletion of these reserves through production, while the contributions from frontier areas will have no real impact until 1990:

> Production from established conventional oil reserves in western Canada will decline substantially over the decade. New discoveries of western conventional oil are expected, but are unlikely to be of sufficient size to offset this decline. New methods of oil extraction such as tertiary recovery will make a growing contribution, but insufficient to make up the shortfall. Major oil sands plants are already in place, providing some 150,000 barrels a day of premium quality oil and others are planned. (p. 93)

As in the EMR paper, the program therefore does not rely on conventional sources of oil but rather on the development of non-conventional oil: tertiary recovery and oil sands. The production of non-conventional oil will increase by 700 per cent in 11 years while conventional oil production will drop by half. After establishing these facts, NEP attempts to bring oil demand into line with supply ten years from now, through a variety of programs.

1.1.3 Summary

Fisher conducted the first econometric study in 1964 and his model was used until 1971. Improvements in oil supply theory were introduced in 1975 by Epple and Eglington in doctoral theses on this topic. Recent studies have integrated the cost of inputs into the model (Epple, Eglington, Uhler) and have approximated an output function.

The major difficulty mentioned by all authors is the lack of data. Another is the special problem of joint production of oil and natural gas. The authors all stress this fact and either disregard it [Eyssel (1973), Erickson, Millsaps and Spann (1971)], or integrate it into their model. Future production is analysed through new discoveries or increases in reserves.

Some authors take into account the cost of inputs (Epple, Eglington, etc.), others adjust for the specific characteristics of the region to be explored -- its size and geological characteristics (Uhler) -- while still others apply only a time-series or cross-section model. Only one author (Uhler, 1979) estimates discoveries for a period that includes the energy crisis.

Obviously oil and gas supply theory is relatively new and no econometric model has gained universal acceptance for estimating the parameters relevant to analysis of this industry. Each author

warns of the inherent limits in application of his model. The following model should also be viewed in this manner, as a contribution to the theory of estimating oil and gas supply. First, however, we will briefly review the Canadian context.

1.2 The Canadian Context

Before 1947, Canadian crude oil met about 10 per cent of domestic demand. The United States, Venezuela and the Arab countries supplied 56, 39 and 5 per cent, respectively, of the total imports required to meet demand. Following the discovery of huge deposits at Leduc in 1947, the Canadian petroleum industry began to compete with imports for domestic sales. Between 1951 and 1957, for competitive reasons, the price of Canadian crude was based on the price of U.S. crude at Sarnia. U.S. crude was gradually replaced by Alberta crude in the Toronto market and *a fortiori* in the western market, where the U.S. was faced with major transportation charges. The eastern Canadian market continued to import cheap Venezuelan crude through Portland, Maine. In 1956, the Suez crisis broke out and the United States found its supplies jeopardized. The U.S. government imposed voluntary import quotas in 1957, then made them compulsory in 1959, for two reasons: to develop a strong U.S. oil industry and therefore to provide reliable sources of supply at all times. During this time (1958-1960), the Suez crisis was resolved and major oil discoveries were made in Libya, Nigeria and Algeria, pushing down the world price for crude.

Import quotas protected U.S. producers from this drop and allowed them to keep domestic prices above world levels. Producers in Canada, however, were forced to lower their prices to meet the competition from imported oil and to protect their market.

U.S. crude was concentrated primarily in Texas, far from the northwestern states; faced with excess capacity, Canadian producers sought U.S. government approval to move into this market. Yet, "while Canadian imports were to be given special consideration, they were not to expand at too rapid a pace. This reduced the industry's incentive to price Canadian crude very much below United States levels." (Bertrand Report, Vol. II, p. 14.) The sword of Damocles -- the possible closing of the U.S. market -- led Canadian companies to sell their crude at U.S. prices in the northwestern states.

But the Americans began to show concern over this situation; Canada was importing Venezuelan crude in the east and exporting its own western oil to the northwestern states: it appeared as though the imported oil was simply moving through Canada and into the U.S. market, although this was actually not the case, as we will see later. The U.S. government "alerted" the Canadian government to this situation: "The United States, in permitting Canadian exports to earn the high American price, felt that, if Canada in turn opened its borders to increased imports, both Canadian consumers and producers would benefit at the expense of

American producer and consumer interests. K. Dam explains American attitudes on this issue: "Canada was regarded as increasing its exports to the United States at the same rate that was increasing its imports from abroad, to the benefit of Eastern Canadian consumers and at no expense to Western Canadian producers. Although no crude oil was actually trans-shipped, the effect on the U.S. quota system was the same as if the crude oil were simply funneled through Canada." (idem, p. 14-15).

In 1961, the Canadian government established the National Oil Policy which made the de facto situation official: west of the Ottawa River, the demand for crude oil was met by Canadian crude; east of the Ottawa River, by imported oil, which therefore could not cross Canada and enter the U.S. market. As demonstrated by the Bertrand Report, this voluntary policy imposed no changes on the oil companies, since they had always found it more profitable to supply eastern Canada with Venezuelan crude (which they themselves imported) than with Canadian crude, while supplying western Canada and Toronto with Canadian crude rather than imported oil, primarily because of the costs of transportation and pipeline construction (no pipeline linked Toronto to the east coast). A sudden drop in the world price for oil, however, could have drastically altered the situation: producers would have halted production in Canada and supplied all of Canada and the U.S. with imported oil. The National Oil Policy therefore assured the Americans that the situation they wished to avoid could never occur.

In 1962, there was a substantial devaluation of the Canadian dollar against the U.S. dollar. This made Canadian crude more competitive in additional U.S. markets. To offset the U.S. government's vulnerability, and forestall any impending suspension of Canadian imports, Canadian producers upped the price in Canadian dollars of their oil. This made the price a few cents cheaper per gallon in eastern Canada than in the West. Subsequent fluctuations in the exchange rate between the Canadian and U.S. dollars, which affected the potential competitiveness of Canadian producers in the U.S. market (or in far fewer instances, of U.S. producers in the Toronto market,), affected the price of Canadian crude oil.

By the early 1970s, U.S. production capacity could no longer meet demand. Canadian exports to the U.S. rose from 202,718,000 barrels in 1969 to 414,429,000 barrels in 1973. Following the oil crisis that same year, the Canadian government began to scale down exports, which dropped to 36,000,000 barrels by 1981.

Chapter 2: The Conventional Approach

2.1 The Preceding Authors

Our review of the literature covered studies and articles that estimated oil and natural gas supply (ONG) through econometric procedures. The table on the following page shows each author's treatment of the various variables.

Except for Erickson, Millsaps and Spann, who use the concept of reserves, and Fisher, who breaks down oil discoveries into three endogenous variables, all of the authors use only discoveries of oil and/or natural gas as endogenous variables in estimating ONG supply. We will use the same approach. Khazoom adds natural gas extensions and revisions to his model but later abandons this additional endogenous variable. The endogenous variable lagged by one year is present as an exogenous variable in the work of all authors except Epple and Uhler. The cost of inputs is used by all except Fisher and Khazoom (Erickson, Millsaps and Spann use the concept of user cost) and the market price by all except Eglington and Epple, who use the concept of the price of reserves in the ground. Only two authors use the success rate as a variable, and each uses it in a completely different way. Determination of the success rate as exogenous or endogenous is tributary to the concept of resource depletion. At the very start of exploration in an untouched region (such as Alberta at the turn of the

Author	Endogenous Variable	Exogenous Variable
Fisher	1. Number of new wells 2. Success rate 3. Average size of reservoir (oil)	A Geophysical teams B Average size of reservoir (natural gas) C Price D 1,2,3 lagged (-1) E Average depth
Khazoom	1. Discoveries (nat. gas) 2. Extensions and revisions	A Ceiling price of natural gas B Price of oil C Price of liquified gases D 1,2 lagged (-1)
Erickson, Millsaps and Spann	1. Oil reserves	A Price B User cost C Dummy variables for districts and for D "Texas Shutdown Days" E 1 lagged (-1)
Epple	1. Discovery of oil (natural gas)	A Price of oil (gas) B Price of reserves C Cost D Technology (cumulative drilling) E Changes in the price ratio
	2. Completed drilling	A Cost B Price of oil and gas C Technology (cumulative drilling)
Eglington	1. Discoveries of oil (natural gas)	A Cost of inputs B Price of reserves (-1) C Success rate (-1) D Anticipated size of reservoir
Uhler	1. Discovery of oil (natural gas)	A Cumulative reserves B Price C Inputs vector

century), the success rate will be strongly linked to the industry's exploration effort. However, as discoveries are made, the resource is depleted and the success rate tends to drop despite the industry's increased exploration effort in this region. If we define the success rate as exogenous, we consider this variable a natural factor beyond the explorer's control; if we define it as endogenous, we indicate that it may be dependent on or strongly influenced by the industry's exploration effort. The concept of resource depletion -- i.e., depletion of the finite quantity of resources available -- first appeared in the early 1970s, seven or eight years after Fisher's article. It is now a very hotly debated topic among natural resources economists. Finally, note that Fisher considers the number of geophysical teams as exogenous. This variable is not a given quantity but rather the result of the industry's decision to undertake a program of exploration, and is therefore endogenous.

In econometrically estimating the supply of ONG discoveries, all of the authors encountered the problem of large swings in this variable. Most got around the difficulty by introducing a lagged endogenous variable as an exogenous variable. As we saw on page 6, this technique makes the interpretation of elasticities (the coefficients of the logarithmic data) difficult. Except for Khazoom, who unsuccessfully attempts to estimate the extensions and revisions in his model, all of the authors estimate the supply of ONG discoveries, and disregard actual production and advanced recovery.

2.2 Construction of Equations

Normally, when we attempt to estimate the supply curve of a good, we estimate the production of this good at various prices. The special characteristic of the natural resources sector is that the good is not produced but extracted. There are two categories of natural resources: renewable (e.g. forests and fish) and non-renewable (e.g. metals). The latter can be subdivided into recyclable and non-recyclable resources. Oil and natural gas are both non-renewable, non-recyclable resources, thus their production (when fully consumed) is irrecoverably lost.

We cannot estimate the ONG supply curve by estimating production of ONG at various prices, because it is very unrealistic to assume the industry is on its supply curve. Canadian production since the industry's birth has been rationed by government to avoid excess supply and a resultant slide in prices. Production therefore does not reflect equilibrium between a supply function and a demand function.

In this work, we will estimate the supply of oil and gas discoveries in Alberta for the 1947-1979 period, through two variables: net back, which is exogenous, and the cumulative endogenous variable lagged by one year. We will therefore estimate the following equation:

$$(1) \quad D_t = \alpha_1 + \alpha_2 RN_{t-1} + \alpha_3 DC_{t-1}$$

where: D_t : appreciated discoveries by year of discovery

RN_t : net back

DC_t : cumulative discoveries

Let us now analyse the components of this equation, beginning with Dt, the discoveries.

The Canadian Petroleum Association (CPA) each year publishes annual discoveries under the heading "Initial Established Reserves by Year of discovery", defined as:

> "Those reserves recoverable under current technology and present and anticipated economic conditions, specifically proved by drilling, testing or production, plus that judgement portion of contiguous recoverable reserves that are interpreted to exist, from geological, geophysical or similar information, with reasonable certainty." (If we exclude production from these reserves, the term "Remaining" must be substituted for "Initial".)

This is an important definition, which we will examine in some detail. Assume, for example, that a reservoir is discovered in 1956 containing estimated recoverable resources of 40 units (million cubic metres) given the 25 per cent recovery rate (the reservoir therefore held a total of 160 units).

Let us now assume that better knowledge of the reservoir in 1958 makes an additional 8 units recoverable. But in 1965, these estimates are found to be overly optimistic and the recoverable quantity is cut from 48 to 43. By comparing CPA data for 1956, 1958 and 1965 (and making the totally unrealistic assumption that no other reservoirs were discovered that year), three figures would appear under recoverable discoveries for 1956 (if we eliminate the publication delays):

	Year of Publication of CPA Statistics		
	1956	1958	1965
Year of Discovery 1956	40	48	43

We must not forget that these reservoirs are huge (some would call them oceans) and that the task of accurately determining the recoverable quantity may continue into part of the extraction cycle.

The expression "current technology" is present to include any technological innovation that improves the recovery rate. The discoveries are adjusted each year to include this change. If a technological innovation in 1968 increases recovery by 10 units, this will give us:

	1956	1958	1965	1968
1956	40	48	43	53

One-third of all discoveries for the year 1980 are attributable to secondary or tertiary recovery, and this is typical of the entire 1947-1980 period. A lack of data prevents us from determining the share of each year's discoveries attributable to enhanced recovery, but we must include enchanced recovery in our data on discoveries. We also note parenthetically that the data on oil discoveries excludes synthetic oil (oil sands).

This whole problem of appreciating discoveries forces us to introduce a factor to account for it in the discovery data. The Alberta Energy Resources Conservation Board analysed this factor for Alberta oil and gas reservoirs. The factor obviously varies between reservoirs and may even prove negative in some cases. However, AERCB has computed an average appreciation factor: after 20 years, 8.09 for oil and 4.971 for natural gas. This factor is rising at a decreasing rate:

	Year After Discovery	Oil Factor	Natural Gas Factor
1979	0	1	1
1978	1	2.48	1.993
1977	2	3.16	2.513
1976	3	3.72	2.822
1975	4	4.20	3.311
1974	5	4.77	3.529
1973	6	5.0	3.758
1972	7	5.79	4.022
1971	8	5.99	4.303
	.	.	.
	.	.	.
	.	.	.
1959	20	8.09	4.971

In our study period, discoveries between 1947 and 1959 have been fully appreciated. The total appreciation factor (X) for 1960-1979, however, must be divided by the annual appreciation factor (xt) and multiplied by the figure obtained through discoveries (Dt) to obtain the appreciated discoveries (DA_t).

$$\frac{X}{x_t} \cdot D_t = DA_t \qquad t = 1959, 1960, \ldots, 1979$$

Taking 1975 as an example, $x_{1975} = 4.20$ and $D_{1975} = 5{,}000$ cm of oil, thus:

$$\frac{8.09}{4.20} \times 5{,}000 = 9.631 \text{ cm of oil.}$$

What role do anticipated economic conditions play? We should expect the increase in the Canadian price for oil to promote a sharp increase in reserves because, as everyone knows, new reserves (and known reservoirs) will become profitable. The CPA includes anticipated economic conditions in its definition of Reserves. Our verification (by comparing annual discoveries for publication years 1967, 1972 and 1980), showed this definition (and the resulting data) to be fully satisfactory.

The endogenous variables to be estimated are therefore the appreciated discoveries of oil and natural gas. What variables influence these discoveries? Can the preceding models provide an

adequate account of the situation in the Canadian ONG industry over the 1947-1979 period? For example, could the Canadian price be the sole variable influencing discoveries? Since 1973, the Canadian price has risen sharply while discoveries of conventional oil and gas have slowed. The geological aspect of the search for new deposits is essential, because no new reservoirs can be discovered if the resource has been depleted. As everyone knows, this variable was ignored by the early economists (Fisher, Khazoom). Most subsequent studies incorporated it by introducing the cost of a discovery. The cost funtion is rising in the natural resources industry, based on the assumption that the largest (or least expensive) reservoirs are discovered first. Since subsequent reservoirs are smaller and/or less accessible, the unit cost increases. Eglington used the success rate and primarily the average size of the reservoir for this same purpose, while Uhler used cumulative reserves. The common thread in all of these variables is the way they link increasing scarcity of the resource to discoveries.

The increasing scarcity of ONG can be directly estimated through cumulative discoveries. A negative relationship between cumulative discoveries and discoveries at time t will indicate resource depletion, while a positive relationship will indicate the availability of large reservoirs. This geological variable will bring the degree of resource depletion into focus. The discovery process is tributary to the fact that large reservoirs are discovered first and small reservoirs last. This process

therefore has a "point of no return" beyond which, as more discoveries are made, fewer deposits remain to be discovered. For oil, we will assume, as did Eglington with the "average size of reservoir" variable and Uhler with the "cumulative reserves" variable, that cumulative discoveries of oil have a negative coefficient, i.e., that an increase in cumulative discoveries of oil triggers a drop in discoveries of oil, or that the point of no return has been passed. For natural gas, however, a positive coefficient would be acceptable because discoveries of large quantities have continued until quite recently.

Our economic variable, which is more complete than price, is the net back, written as:

$$RN = P - C - I - T - R \qquad (2)$$

where RN : net back

P : market price of the resource

C : operating costs

I : income taxes paid

T : taxes paid

R : royalties paid

RN includes the recent, major changes in ONG fiscal policy. RN is used by the industry to measure the impact of any change in these variables. Any variation in RN has effects (delayed in the case of an increase and immediate in the case of a decrease) on the

quantity of ONG discovered. Naturally, we will assume that net back has a positive coefficient, i.e., that an increase in net back induces an increase in discoveries.

To take into account the fact that RN is a flow (because it extends over the productive life of a well), we ought to divide it by a discount factor. Since this factor has to be assumed constant here, however (the specific factors for each year will be determined in another study), the results obtained (coefficients, Student, R2, DW) would be the same, except for the constant term. RN is therefore a satisfactory proxy for measuring the value of discoveries in the ground.

We therefore have two variables for estimating discoveries of ONG: first, an economic variable -- net back, a proxy that measures the value per unit in the ground as estimated by the industry, after deducting relevant costs and taxes -- and, second, a geological variable that determines the degree of resource depletion.

Let us return to the equation that will estimate discoveries of ONG based on the exogenous variables of net back (RN) and cumulative discoveries (DC):

$$(1) \quad D_t = \alpha_1 + \alpha_2 \, RN_{t-1} + \alpha_3 \, DC_{t-1}$$

The delayed effect of RN is estimated by an "Almon distributed lag" which constrains the influence of a variable to a quadratic form, here in the second degree, lagged by three years. We also tried, unsuccessfully, approximating the lagged effect with simple lags (t-1, t-2).

A serious problem arises here, because we are dealing with a joint product: most oil reservoirs also contain natural gas, although the opposite occurs much less frequently. The CPA provides complete statistics on discoveries of either resource, so it is easy to subdivide the endogenous variable and thus by corollary, the lagged endogenous "cumulative discoveries" variable, but for the exogenous "net back" variable, there are no disaggregated data on royalties, taxes and income taxes. Those authors who chose the cost of inputs as an exogenous variable encountered the same problem, and solved it by weighting the total costs by each resource's share in a variable measuring the discovery intent (generally the number of wells drilled).

The CPA provides data on the earnings from sales of the two resources. At this stage in our work, we will make the fairly mechanical assumption that each product's share of total earnings equals that product's share of total costs (royalties , operating costs, taxes, etc.). Equation (1) is thus divided in two:

$$1' \quad DGN_t = \alpha_1 + \alpha_2 \, RNG_{t-1} \, \alpha_3 \, DCG$$

where $RNG = P_G - \delta CT$, δ being the share of total earnings from sales of natural gas and CT the total costs.

$$1'' \quad DP_t = \alpha_1 + \alpha_2 \, RNP_{t-1} + \alpha_3 \, DCP_{t-1}$$

where $RNP = P_p - (1-\delta)CT$

2.3 Data, Estimations and Results

The endogenous variables to be estimated are discoveries of oil and/or natural gas between 1947 and 1979 in Canada. The exogenous variables are the cumulative discoveries and RN for each resource.

Except for the data on income taxes paid, obtained from Statistics Canada Catalogue No. 61-208, the CPA 1980 statistics manual provided all the necessary data for our analysis. The degree of aggregation of the variables forming RN raised a problem which, as already mentioned, we solved by assuming that a resources's share of the total sales for both resources equals that resource's share of the costs involved in its production.

The ordinary least squares method was used for the estimation. All of the data were converted into logarithms, and produced the following results and Student values:

$$(1') \quad \ln DGN_t = 5.07 - .83 \ln RNG_{t-1} - .017 \ln DCGN_{t-1}$$
$$(6.27)(-1.73) \qquad (-.11)$$
$$R^2 = .18 \quad DW = .78$$

$$(1'') \quad \ln DP_t = -23.7 + 6.24 \ln RNP_{t-1} + 1.53 \ln DCP_{t-1}$$
$$(-3.11) \quad (4.57) \qquad (2.62)$$
$$R^2 = .61 \quad DW = .77$$

The natural gas equation is very disappointing for several reasons: The R2 and DW values are very small, the coefficient of cumulative discoveries is non-significant and the coefficient of net back has the opposite sign to that expected.

The results for oil are more encouraging. Except for a very small DW value, all other values appear valid: the coefficients are significant and the R2 figure is high. A one percentage point increase in net back for oil causes oil discoveries to rise by 6.24 points. However, the coefficient of cumulative discoveries is contrary to our expectations. In fact, a positive coefficient greater than one indicates a rising oil discovery rate.

Finally, our coefficient of net back proves to be twice as large as the greatest equivalent coefficient in the literature (Fisher's price elasticity of 2.85).

The very dubious validity of these results convinced us to conduct further research, involving experiments that failed:

integrating prices and costs as exogenous variables, and using only a one-year lag (rather than an Almon lag) for net back. The coefficients were either non-significant or had the wrong sign, or else the R2 and DW values were too small.

This could mean that the assumption on the division of costs between oil and natural gas is inaccurate.

Since oil and gas are joint products for discovery purposes, we will now take an aggregate approach. We must, however, assume perfect substitutability between the two types of discoveries, i.e., that explorers have no preference for discovering either oil or gas. Justification for this assumption increased as natural gas exports to the U.S. grew in the 1960s and 1970s.

This combination requires a new, very common concept to estimate future demand for energy: the petajoule (10^{15} joules). Net back is obtained in the same manner, with no need to break down the data. The results are:

$$(1) \quad \ln DT_t = \underset{(-1.19)}{-1.72} + \underset{(7.30)}{1.98} \ln RNT_{t-1} + \underset{(2.27)}{.26} \ln DCT_{t-1}$$

$$R^2 = .75 \quad DW = 1.25$$

This is an improvement over the previous results. Except for the constant term, the coefficients are significant: a one percentage point increase in net back produces an increase of

almost two percentage points in discoveries. The positive sign of the coefficent for cumulative discoveries indicates that the resources do not appear to be depleted, since an increase in cumulative discoveries does not produce a decrease in discoveries.

The only cloud in the picture is the weakness of DW which probably means that one or more variables are missing from the specification of discoveries. The reader will recall that we have a geological variable (cumulative discoveries) and an economic variable (net back). Referring to other authors, we find that all except Fisher and Khazoom have also included an input variable, which our approach lacks. When we add the exploration effort variable (with an almon lag), expressed as the depth of exploratory wells in metres, we obtain:

$$\ln DT_t = \underset{(-1.72)}{-2.76} + \underset{(7.01)}{2.04} \ln RNT_{t-1} + \underset{(.86)}{.26} \ln DCT_{t-1} - \underset{(.37)}{.12} \ln X$$

$$R^2 = .75 \qquad DW = 1.33$$

where X = depth of exploratory wells in metres.

The R2 and DW values improve, but the coefficient for exploration effort is non-significant and the coefficent for cumulative discoveries becomes non-significant. The introduction of the exploration effort variable has failed to improve the results. We must therefore re-examine the theory itself and redirect our research.

Chapter 3: A New Approach

3.1 Introduction

As everyone knows, a firm's supply curve is its marginal cost curve. Since the total supply curve for any commodity is the horizontal sum of the individual supply curves of the individual firms producing that commodity, at the aggregate level it is therefore the total marginal cost curve.

Since we are unable to estimate the supply curve for oil and gas discoveries through the procedure described in the preceding chapter, we will estimate it in this chapter through the marginal cost curve.[1] A. MacFayen laid the foundation for this theory, as we later discovered. The use of cumulative discoveries as an exogenous variable does, however, seem to be new.

The first question that comes to mind is obviously what costs enter into the "production" of an ONG discovery. Because we are interested in ONG discoveries rather than actual extraction, all of the costs involved in discovery must be taken into consideration. Let us now define these costs in detail, developing the theory underlying estimation of the marginal cost curve and the choice of variables, and then analyse the results obtained.

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1 I must acknowledge the assistance of T. Schweitzer and N. Swan in the development of this chapter.

3.2 The Marginal Cost Curve

3.2.1 The Theory

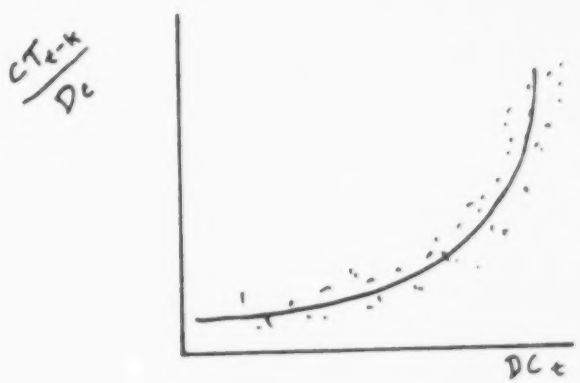
This chapter will focus on the exploration and development expenditures that were deliberately sidestepped until now. As pointed out in the preceding section, these expenditures have risen dramatically in recent years. We will define "total costs" as all exploration and development expenditures made by the industry in Alberta. The distinction between exploration expenditures and development expenditures is this: the first cover costs incurred in producing a discovery. Once a reservoir is discovered, additional wells are drilled, findings are analysed and a broad range of scientific tests are conducted to determine the quantity and quality of the resource and its components. These development expenditures are included in the costs, because discoveries appreciate through develoment expenditures, as mentioned earlier. These expenditures occur in the two or three years immediately following a discovery in order to determine the precise quantity of the resource in the reservoir. If one-third of these expenditures are assigned to each year in a three-year period, their sum ($.33\ C_t + .33\ C_{t+1} + .33\ C_{t+2}$ for year t) differs little from the results if we do not include such dispersion, so we will take the latter course. We will define "discoveries" as discoveries of oil plus discoveries of gas, measured in petajoules as before.

All of these data are available for the 1947-1981 period. Of course, the data on discoveries had to be revised with the adjusted appreciation rates for all years to include the years 1980 and 1981.

Our cost variable must take into account the increasing scarcity of resources: we therefore must determine the cost per unit, by dividing the total cost for each year by the number of discoveries made in the same year or subsequent years, the latter to allow for the fact that an exploration program does not necessarily produce immediate results -- the same allowance made for net back in the preceding section.

Plotting the total costs divided by annual discoveries (henceforth CT_{t-k}/D_t) and discoveries on a chart produces only a swarm of scattered points. To obtain a curve that we can estimate econometrically, we must use the cumulative discoveries variable (henceforth DC_t) which is already known. When this forms the abscissa, ideally, each point should shift to the right from its initial position, as in Chart 1 below.

Chart 1



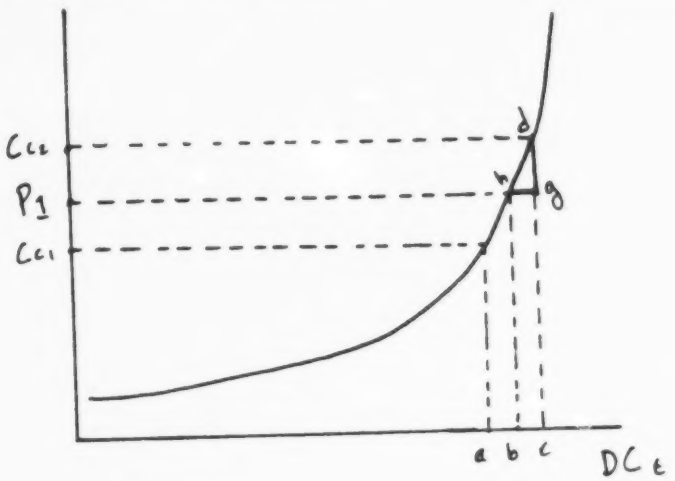
Is this a marginal cost curve? Yes, it is an average patn marginal cost curve for discoveries, by petajoules, because it represents the actually observed values for incremental annual costs per unit discovered for each year over the entire period. Uhler (1971) demonstrated that the natural resources industry faces a growing marginal cost curve, particularly in a specific region such as Alberta. The oil and gas industry is no exception to this rule, so we can assume that the industry's marginal cost curve is like that shown in Chart 1.

If we can successfully estimate this curve, the results will provide a wealth of information, such as the cost of discovering the next reservoir. We might even be able to estimate the quantity of "profitable" discoveries in Canada for a given world price. We could then continue exploration until the marginal cost per unit discovered equals the world price for oil, and then cut off funds when the cost exceeds this level. This approach will only be valid, however, if we exclude all costs that are simply a transfer. The drilling rights to land are one such cost because they do not represent a consumption of inputs by the firm, and are therefore excluded. For greater accuracy, we must also add to the marginal cost (or subtract from the price) transportation and operating costs, and continue exploration until

$$(1) \quad \frac{CT_{t-K}}{D_t} + CTO_t = P_e$$

where CTO is the transportation and operating costs, and Pe is the weighted price in BTUs for oil and gas in western Canada, i.e., the number of dollars that can be obtained for discovering and developing one cubic metre of oil or 1,000 cubic metres of natural gas. This quantity of "economic" discoveries is illustrated in Chart 2.

Chart 2



Let us assume that the discovery cost Cc (including CTO) is at Cc1 and the world price is at PI; Canada can profitably discover quantity ab, i.e., until Cc = PI. At this point, Canada can purchase an almost unlimited quantity on the world market (assuming that the Canadian market is too small to influence PI). If, however, exploration continued for whatever reason -- a self-sufficiency policy, for example -- and we reached Cc2, we would discover quantity bc. The policy would therefore carry a cost represented by triangle dhg, or in mathematical terms, by the expression:

$$\int_{d}^{c} F(DC)\, d\, DC - P_I (c-b)$$

where F(DC) is the equation of the curve.

Another useful fact can be drawn from estimation of the curve in Chart 1: since this curve is non-linear (costs increase at a growing rate), it is possible to estimate an "economic" limit to cumulative discoveries. The economic limit of discoveries is reached when the shift to the right becomes very small compared with the shift upwards; beyond this point, additional resources can only be discovered at an infinite cost.

The equation to be estimated is therefore:

$$(2) \qquad \frac{CT_{t-k}}{D_t} = F(DC_t)$$

where CT = exploration and development expenditures less drilling rights
DC = appreciated cumulative discoveries

3.2.2 Results

We tested (2) with a specification using the logarithms of the variables. All combinations of k = 0,1,2,3, and n = 1,2,3,4 were attempted. The following equation produced the best results (R2, DW, Student statistics):

$$(3)\quad \ln (CT_{t-3}/ D_t) = -10.44 + 1.47 \ln DCT_t$$
$$(-7.42)\ (8.26)$$
$$R^2 = .68 \quad DW = 1.93$$

This is quite interesting: a one percentage point increase in cumulative discoveries produces a 1.47 percentage point increase in costs (lagged by three years); the marginal cost curve is therefore rising at a growing rate. This equation enables us to conduct a few simulations, remembering, of course, that discoveries are in petajoules, or the equivalent of one cubic metre of oil or 1,000 cubic metres of natural gas.

To determine the discovery cost in 1981, resulting in discovery in 1984, we simply insert into the equation the figure on real cumulative discoveries, in this case 4,671 petajoules, and solve the equation for

$$(3)\quad \ln (CT_{t-3}/D_t) = -10.44 + 1.47 \ln (4671)$$
$$CT_{t-3}/D_t = \$7.24$$

Thus, today's cost would be $7.24 (or $7.24 X CPI = $19.00 in 1982 dollars) for 1,000 cubic metres of natural gas in 1984. We will now estimate the discoveries at a specific price or net back. Assuming oil is the preferred resource, we will estimate the quantity of petajoules discovered by computing the discovery cost equal to the wellhead price for new oil ($260/cm or $99.03/cm in 1971 dollars). If we use a net back of 50 per cent (including

transportation and operating costs) we obtain $49.52. If we assume that the industry ploughs all earnings back into exploration and development, we obtain:

$$(3) \quad \ln (49.52) = -10.44 + 1.47 \ln DCT_t$$

$$DCT_t = 17,270 \text{ pétajoules.}$$

According to our equation, we have only discovered one fourth of all the resources in Alberta to date (4,671/17,270), which is surprising. When we analyse the real data, we find that the years with the highest exploration and development expenditures are excluded (1981 = t; 1980 = t-1; 1979 = t-2) because they produce the least valid statistics (R2 and DW). Furthermore, we tested the equation with the price for new oil, which is higher than the price for new gas.

Let us repeat that we find these results surprising because they imply that very large discoveries will be made in the very near future.

Since this result is the "synthesis" of the two resources (oil and gas), the results of our equation only permit us to conclude that quantitites of oil and/or natural gas remain to be discovered in Alberta. There is no way of determining which resource, if either, is more abundant than the other.

We will estimate the same equation, but with a breakdown between the resources.

3.3 The Disaggregated Curve

3.3.1 Discovery Intent

What share of exploration and development costs should be assigned to oil and to natural gas? We have already attempted to divide these costs by the share of total earnings generated by each resource, but the results proved unsatisfactory.

Like most authors, we will use the discovery intent to assign costs to oil and gas. The sole source of such data in Canada is P. Eglington's doctoral thesis, but his series ends in 1970, so we must estimate intents for the 1970-1981 period. This will be accomplished by postulating a constant success rate (i.e., the number of wells with oil intent over the number of wells with oil discovery). Our procedure will be as follows:

	Total Wells Drilled	Wells Drilled For Oil	Wells Drilled with Oil Discovery
		(number of wells)	
1947	T_{47}	X_{47}	Y_{47}
1948	T_{48}	X_{48}	Y_{48}
.	.	.	.
.	.	.	.
.	.	.	.
1970	T_{70}	X_{70}	Y_{70}
1971	T_{71}	n.a.	Y_{71}
1972	T_{72}	n.a.	Y_{72}
.	.	.	.
.	.	.	.
.	.	.	.
1980	T_{80}	n.a.	Y_{80}
1981	T_{81}	n.a.	Y_{81}

n.a. : not available

Let i = 1947, 1948, ... 1970 and j = 1971, 1972, ... 1981

we have $\frac{\Sigma\ Yi}{\Sigma\ Xi} = .211$ from Eglington (1975), we are looking for X_j, and we know Y_j. We obtain X_j by postulating

$$X_j = Y_j/.211$$

We now have X_j and can find G_j with

$$T_j - X_j = G_j$$

where G = wells drilled for gas.

This exercise is valid if we make two crucial assumptions: first, drilling for either oil or gas costs the same and second, dry wells are distributed between the two resources in the same proportion as successful wells. We have disregarded the possibility of estimating Xj by regressing X over Y for the 1947-1970 period.

3.3.2 Breaking Down the Polynomial

We still wish to estimate the disaggregated marginal cost curve. This has already been defined by the annual exploration and development expenditures broken down by discovery intent, divided by the annual discoveries of one resource or the other. Once again, the equations to be estimated are:

$$(4) \quad CP_{t-k}/DP_t = F(DCP)$$

$$(5) \quad CG_{t-k}/DG_t = F(DCG)$$

where CP_{t-k} and CG_{t-k} are the costs associated with oil and gas respectively.

These equations were obtained from the aggregate equation at the start of the chapter:

$$(2) \quad CT_{t-k}/D_t = F(DC)$$

Except for the fact that the equations are broken down, the exercise is identical to that at the start of this chapter.

3.3.3 Results

The results are all given in logarithms:

$$(6)\quad \ln (CP_{t-1}/DP_t) = -10.66 + 1.765 \ln DCP_t$$
$$(-5.53)\quad (6.56)$$
$$R^2 = .56 \qquad DW = 1.26$$

$$(7)\quad \ln (CG_{t-3}/DG_t) = 7.81 + 1.187 \ln DCG_t$$
$$(-6.06)\quad (6.60)$$
$$R^2 = .58 \qquad DW = 1.78$$

The coefficients have the proper sign and are all significant. The R2 and DW values are satisfactory. The most interesting information emerging from these results is that the costs for oil and natural gas increase at a growing rate.

By inserting the real figure for 1981 cumulative discoveries into the two equations, we obtain the current cost of discovery and development for each resource:

Oil	Natural Gas
(6) $\ln x = -10.66+1.765\ln(2067)$	(7) $\ln y = -7.81+1.187\ln(2604)$
$x = \$16.67/cm$	$y = \$4.60/000$ c.m.
or \$43.77 in 1982 dollars	or \$12.20 in 1982 dollars

Let us now do another interesting exercise. Consider the point on our fitted marginal cost curve where marginal cost is equal to the actual 1982 net back the industry received. This netback was about 50% of the present Canadian price. For use in our equations, this netback must be converted to 1971 dollars. Thus, we are interested in the point on our marginal cost curve where MC = (1982 price ÷ CPI) x net back, for each of oil and gas. This gives us:

Oil: (\$260 ÷ 2.6254) x .5 = 49.52 in 1971 dollars

Natural Gas: (\$169. ÷ 2.6254) x .5 = 32.19 in 1971 dollars

Given the marginal costs, we can now use the equations to estimate the ultimate cumulative discoveries implied by present oil and gas prices. For oil, we have:

$$\ln (49.52) = -10.66 + 1.765 \ln DCP_t$$

DCP = 3830 million cubic metres of oil

and for gas:

$$\ln (32.19) = -7.81 + 1.187 \ln DCG_t$$

DCG = 13,418 billion cubic metres of gas

Since the cumulative discoveries of oil in 1981 amounted to 2,067 million cubic metres, the model in this form (we develop it further below) shows that about one half of Alberta's total oil resources, or 1763 million c.m. remain to be discovered and for gas 10814 billion c.m. The actual cost data underlying these results may be of interest. They are summarized in Table 1 following. It shows the real growth of discovery and development costs for both resources. We have grouped the expenditures into roughly equal amounts of $1.5 billion, with the scarcity of the resource becoming evident through dimishing discoveries as these expenditures are made. We see that with constant expenditures, oil discoveries occur at a more rapidly declining rate than natural gas discoveries, a fact supported by the results of the equation.

Table 1

Growth of Discovery and Development Costs in Alberta, for Each $1.5 billion of Expenditure

	CUMULATIVE COSTS	COSTS	DISC.	COST/CM
		OIL		
1947-56	1580	1580	1374	$1.15
1957-62	2989	1409	502	$2.81
1963-67	4364	1375	320	$4.30
1968-73	5885	1521	80	$19.01
1974-78	7600	1715	92	$18.64
1979	8637	1037	38	$27.29
1980	10167	1530	10	$153.00
1981	11697	1530	49	$31.22
		NATURAL GAS		
1947-65	1548	1548	1509	$1.03
1966-73	3222	1674	411	$4.07
1974-76	4560	1338	278	$4.81
1977-79	6120	1560	281	$5.55
1980-81	6977	857	125	$6.86

Source Author's compilation from the CPA 1982 statistics manual

3.3.4 Development of the Theory

3.3.4.1 The Same Cost Specification

Given the disappointing DW value in the oil equation, we decided to develop the analysis further. Our first step was to add a quadratic term to the equations. With this change, the optimal lag turned out to be different and shorter thus partly resolving the problem of omission, mentioned above, of the very high end-of-period costs, with the following results:

$$\ln(CP_{t-1}/DP_t) = 19.336 - 7.42 \ln DCP_t + .69 \ln^2 DCP_t$$
$$(1.01) \quad (-1.27) \quad (1.58)$$

$$R^2 = .58 \qquad DW = 1.42$$

$$\ln(CG_{t-1}/DG_t) = 14.6478 - 5.62 \ln DCG_t + .51 \ln^2 DCG_t$$
$$(3.90) \quad (-4.55) \quad (5.17)$$

$$R^2 = .72 \qquad DW = 2.27$$

There is an improvement in all statistics except the Student values in the oil equation, and the improvement in the gas equation is spectacular. It is interesting again to obtain the ultimate cumulative discoveries of both resources, with these revised equations. To do this, we solve:

$$\ln(49.52) = 19.336 - 7.42 \ln DCP_t + .69 \ln^2 DCP_t$$

$$\ln (32.19 = 14.6478 - 5.62 \ln DCG + .51 \ln^2 DCG_t$$

For oil, DCP = 2644 million cm and for gas, DCG = 4332 billion cm. By subtracting the 1981 cumulative discoveries, we now obtain 577 million cm for oil and 1728 billion cm for natural gas.

These results were based on actual oil and gas prices in 1982. However, prices may well go beyond these levels in future years and discoveries will therefore be higher than those just estimated. For example, the NEB has considered in its forecasting analysis three possible year 2000 prices for oil. They were, in 1982 dollars, $189/c.m. (the base case under the NEP) and two higher values (the "modified" case, $239 and the high case $300). Let us see how much oil might be discovered at these prices using our equation. It will be interesting simultaneously to compare our results with the NEB forecasts at these same prices.

To do so, we first convert the three NEB prices to numbers we can insert directly into our oil equation. As explained earlier, that means finding out what these prices would imply for netbacks and also, incidentally, expressing them in 1971 dollars. We assumed that the netback would be between 35% and 45% of the prices. Thus, using the $189 price as an example, we insert into our equation two alternative values:

($189 ÷ 2.6254) x .35 = $25.20 in 1971 dollars

and ($189 ÷ 2.6254) x .45 = $32.40 in 1971 dollars

Proceeding in that way, we obtained our forecasts, which are shown above, with the NEB forecasts in table 2:

Table 2

Price assumption in current dollars, NEB	NEB forecast, Alberta (10^6 c.m.)	Our forecasts (10^6 c.m.) Netback 35%	Netback 45%
Base ($189 c.m.)	259	96	268
Modified ($239 c.m.)	259	257	436
High ($300 c.m.)	446	418	605

As can be seen, much depends on the netback assumption. In general, however, our results are more optimistic than the NEB, particularly so if netbacks are relatively generous.

Comparisons with the NEB can also be made for natural gas. At the time of writing this document, the latest estimations of potential for gas discoveries in Alberta are contained in Table 3-2 on page 15 of the NEB paper "Reasons for Decisions in the Matter of Phase II - License Phase - and Phase III 0 the Surplus Phase - of the Gas Export Omnibus Hearing, 1982" published in January 1983. The 1983-2000 forecasts for Alberta are 37.9 exajoules. Our equation gives, at the gas price used a result of 1728 billion c.m. When converted to exajoules, this becomes 67.4 exajoules (1728 x .039), almost double the NEB forecast.

3.3.4.2 A Different Cost Specification

In the equations used so far, the distribution of exploration intents has been linked as much to exploration expenditures as to development expenditures, primarily because no data exist from 1947 to 1954 on development wells associated with oil or gas,

while Eglington's thesis provides all relevant data for exploration costs. But there is no correlation between exploration intent and development intent (we estimated a regression for the period available, 1955-1981). It would therefore be preferable to break down the data on development expenditures not by exploration intent but rather by the distribution of development wells between oil and gas. (We could also use the quantity of drilling in metres associated with each resource, but the statistics are poorer.) Data were available from 1954 to do this, but not from 1947-54. To cope with these years we note that, the data on development wells leading to discovery of oil or natural gas reveals that in the 1950s, oil accounted for about 90 per cent of discovery intents, with peaks of 96 per cent in the first years. We therefore set the proportion at 95 per cent for 1947-1954, which must be very close to the real figure (the first gas pipeline was built between 1956 and 1958).

The revised distribution of costs between oil and gas produces the following results:

$$\ln(CP_t/D_t) = 7.08 - 3.32 \ln DCP_t + .36 \ln^2 DCP_t$$
$$(.98) \quad (-1.41) \qquad (1.93)$$
$$R^2 = .54 \qquad DW = 1.55$$

$$\ln (CP_{t-1}/D_t) = 20.51 - 7.98 \ln DCG_t + .72 \ln^2 DCG_t$$
$$(4.99) \quad (-5.89) \qquad (6.65)$$
$$R^2 = .80 \qquad DW = 1.94$$

These are much improved fits. Let us now repeat the previous exercise. First, we find ultimate discoveries at present oil and gas prices by inserting 49.52 and 32.19 in the equations, as before:

$$\ln (49.52) = 7.08 - 3.32 \ln DCP + .36 \ln^2 DCP$$

$$\ln (32.19) = 20.51 - 7.98 \ln DCG + .72 \ln^2 DCG$$

Solving the equations, we find that 1027 million c.m. (3094 -2067) of oil and 932 billion c.m. (3536 - 2604) of gas remain in Alberta. This is twice as much oil and half as much gas as with the earlier cost specification. The cost concept used therefore matters a lot. It is our view that both logic and the better fit favor the cost concept used in this section. With this in mind, it becomes important to find what it implies under the NEB prior scenarios. Repeating the kind of analysis done above, we obtain the results shown in Table 3.

Table 3

Price assumptions in current dollars NEB	NEB forecast Alberta (10^6 c.m.)	Our forecast (10^6 c.m.) Netback 35%	Netback 45%
Base ($189 c.m.)	259	275	537
Modified base ($239 c.m.)	259	519	801
High ($300 c.m.)	446	773	1 073

These results are considerably higher than our previous results. Our earlier conclusion therefore is reinforced: we are more optimistic than the NEB on future discoveries of oil in Alberta. At this base case price, for example, and at the middle of our netback range, we foresee over 50% more than NEB. The gap widens, the higher are prices and the more generous are netbacks.

On gas, our new estimate virtually coincides with NEB, 36 exajoules versus NEB's 38.

SUMMARY AND CONCLUSIONS

The review of the literature allowed us to draw up a list of variables that determine the supply curve for oil and natural gas in Alberta. The variables considered were net back, an economic variable, and cumulative discoveries, a geological variable. The results of the econometric estimations were disappointing. To explain aggregate ONG supply, a new approach based on estimating the marginal cost curve including exploration and development expenditures was attempted, with a logarithmic equation. The results were found to be inconclusive, so we attempted to break down the costs by discovery intent, while retaining the logarithmic estimation.

First we added a quadratic term to the equations estimated, and then modified the cost concept in a way we think plausible. The results we consider the best imply that there is at least 50% more conventional oil remaining to be discovered in Alberta then the NEB has forecast. There may be more, depending on prices and the degree of generosity of netbacks. On natural gas, our forecasts coincide almost exactly with NEB.

In conclusion, we can therefore state that our forecasts are more optimistic for oil than those of the NEB, but the same for gas. Because the equations fit is not perfect however, readers should judge for themselves how much confidence can reasonably be placed in these results. The model does show rather conclusively that price affects the quantity of oil and natural gas available

in a more or less continuous fashion, and by the same token, so does the degree of generosity of netbacks.

Appendix 1

	Appreciated discoveries (1979 publication)			Net backs $1971		
	Oil (thousands of c.m.)	Natural gas (trillions of c.m.)	Total petajoules (10^{15}BTU)	Oil ($/c.m.)	Natural gas ($/000 c.m.)	Total ($/$10^{6}$BTU)
1947	60	47	107	25.46	2 99	14.22
1948	143	18	161	23.52	1.38	12.45
1949	65	35	100	22.00	1.73	13.89
1950	68	38	106	23.42	1.73	16.19
1951	73	27	100	17.51	1.58	13.97
1952	145	205	350	15.91	2.04	12.45
1953	293	138	431	16.68	2.42	13.83
1954	58	93	151	17.09	2.09	13.76
1955	30	37	67	16.04	2.63	13.60
1956	32	118	150	16.07	2.28	13.60
1957	241	103	344	16.46	2.35	13.43
1958	47	116	163	14.30	2.25	10.59
1959	242	160	402	13.91	2.37	10.33
1960	10	50	60	12.67	2.16	8.85
1961	13	160	173	12.74	2.85	9.03
1962	35	76	111	11.87	2.85	7.74
1963	34	28	62	12.03	2.96	7.67
1964	74	25	99	12.42	3.11	7.76
1965	112	44	156	12.50	3.04	7.61
1966	72	16	88	12.04	3.10	7.57
1967	44	76	120	10.42	2.82	6.72
1968	32	24	56	10.36	2.74	6.55
1969	18	71	89	10.21	2.80	6.46
1970	10	22	32	10.77	2.99	6.81
1971	8	47	55	11.14	2.78	6.85
1972	7	52	59	10.53	2.67	6.60
1973	10	38	48	11.70	2.65	7.43
1974	5	123	128	14.04	3.13	8.80
1975	3	71	74	13.43	5.06	9.07
1976	37	127	164	14.16	7.42	10.44
1977	74	89	163	16.50	9.36	12.47
1978	23	87	110	17.71	9.89	13.32
1979	40	75	115	17.54	10.49	13.71

Appendix 2

	Oil			Natural Gas		
	Costs ($1971) ($millions)	Appreciated discoveries (1981 publication) (millions c.m.)	Expenditure per discovery $/c.m.	Costs ($1971) ($millions)	Appreciated discoveries (1981 publication) (trillions c.m.)	Expenditure per discovery $/Mc.m.
1947	24.87	60	.41	11.70	46	.25
1948	61.03	141	.43	26.15	18	1.45
1949	101.59	61	1.66	35.69	38	.93
1950	150.15	69	2.17	30.75	43	.71
1951	158.14	74	2.13	64.59	29	2.22
1952	191.57	148	1.29	67.30	212	.31
1953	185.97	295	.63	79.70	143	.55
1954	173.44	65	2.66	74.33	96	.77
1955	237.87	30	7.92	92.50	37	2.50
1956	294.92	33	8.93	103.62	120	.86
1957	236.56	199	1.18	78.85	102	.77
1958	250.91	50	5.01	97.57	115	.84
1959	232.72	195	1.19	86.07	158	.54
1960	237.16	10	23.71	116.81	51	2.29
1961	260.27	13	20.02	146.40	161	.90
1962	191.28	35	5.46	122.29	79	1.54
1963	236.85	36	6.57	106.41	31	3.43
1964	259.85	74	3.51	91.29	30	3.04
1965	257.14	103	2.49	115.53	50	2.31
1966	243.70	66	3.69	143.13	15	9.54
1967	376.92	41	9.19	106.31	72	1.47
1968	351.67	29	12.12	117.22	20	5.86
1969	323.02	16	20.18	152.01	75	2.02
1970	248.02	8	31.00	258.15	29	8.90
1971	271.44	12	22.62	250.56	51	4.91
1972	130.66	7	18.66	353.26	61	5.79
1973	152.04	8	19.01	338.42	38	8.91
1974	159.94	6	26.66	411.26	119	3.46
1975	163.73	2	81.87	421.02	69	6.10
1976	209.51	8	26.19	628.53	90	6.98
1977	331.08	15	22.07	588.58	84	7.01
1978	569.69	61	9.34	642.41	92	6.98
1979	1036.70	38	27.28	487.87	105	4.64
1980	1530.00	10	153.00	565.90	50	11.30
1981	1529.70	49	31.21	291.36	75	3.88

Oil plus natural gas

	Total costs ($1971) ($millions)	Appreciated discoveries (1981 publication)	Expenditure per discovery
1947	36.58	106	.34
1948	87.18	159	.54
1949	137.29	99	1.38
1950	180.90	112	1.61
1951	222.73	103	2.16
1952	258.88	360	.71
1953	265.67	438	.60
1954	247.77	161	1.53
1955	330.37	67	4.93
1956	398.54	153	2.60
1957	315.42	301	1.04
1958	348.48	165	2.11
1959	318.80	353	.90
1960	353.97	61	5.80
1961	406.67	174	2.33
1962	313.57	114	2.75
1963	343.26	67	5.12
1964	351.15	104	3.37
1965	372.67	153	2.43
1966	386.83	81	4.77
1967	483.24	113	4.27
1968	468.89	49	9.56
1969	475.03	91	5.22
1970	506.17	37	13.68
1971	522.00	63	8.28
1972	483.92	68	7.11
1973	490.47	46	10.66
1974	571.20	125	4.56
1975	584.75	71	8.23
1976	838.03	98	8.55
1977	919.66	99	9.28
1978	1212.10	153	7.92
1979	1524.60	143	10.66
1980	2095.90	60	34.93
1981	1821.00	124	14.68

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